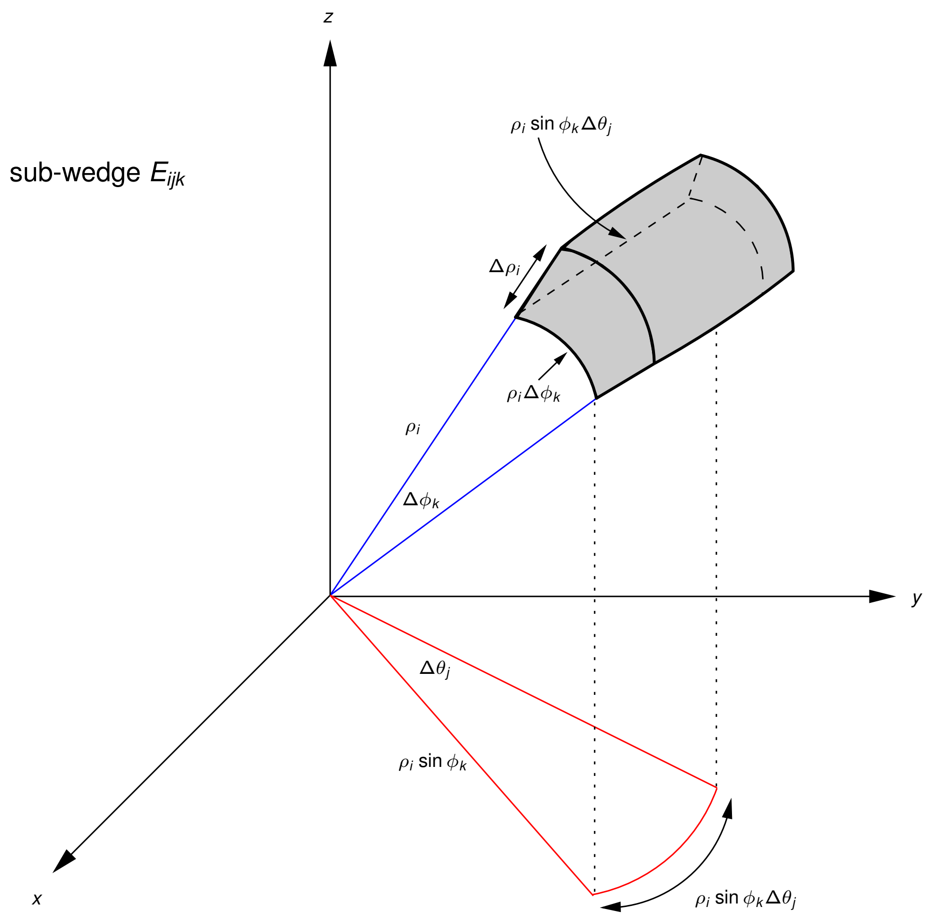
1 Spherical Coordinates

Consider the spherical wedge

$$E = \{(\rho, \theta, \phi)_{\text{spherical}} \ : \ a \le \rho \le b, \alpha \le \theta \le \beta, c \le \phi \le d\}$$

where $\beta - \alpha \le 2\pi$. We wish to compute $\iiint_E f(x, y, z)\, dV$.

Partition the ρ, θ and ϕ intervals into ℓ, m and n subintervals, respectively, which defines small sub-wedges:



Here E_{ijk} has approximate volume

$$V_{ijk} \approx (\Delta\rho_i)(\rho_i \sin\phi_k \Delta\theta_j)(\rho_i \Delta\phi_k) = \rho_i^2 \sin\phi_k \Delta\rho_i \Delta\theta_j \Delta\phi_k$$

Consequently,

$$\iiint_E f(x, y, z)\, dV = \int_{\phi=c}^{d} \int_{\theta=\alpha}^{\beta} \int_{\rho=a}^{b} f(\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho \cos\phi)\, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$