Question 1: A right triangle has perpendicular sides of length $b = 5$ and $h = 12$ m. Let $f(b, h)$ represent the total (i.e. the sum) of the area of the triangle and the length of its hypotenuse. If b is increased by 0.1 and h is decreased by 0.2, use differentials to estimate the resulting change in f.

[5]

Question 2: Suppose $P = \sqrt{u^2 + v^2 + w^2}$ where

$$u = xe^y, \qquad v = ye^x, \qquad w = e^{xy}$$

Determine $\dfrac{\partial P}{\partial x}$ when $x = 0$ and $y = 2$.

[5]

Question 3: Find all points on the paraboloid $y = x^2 + z^2$ at which the tangent plane is parallel to the plane $x + 2y + 3z = 1$.

[5]

Question 4: The temperature at point $P(x, y, z)$ is given by

$$T = \frac{e^{-3x^2-4y^2+4z^2}}{4}$$

where x, y, z are in metres. Find (i) the maximum rate of increase of temperature at the point $P_1(2, -1, 2)$ and (ii) the direction in which it occurs (state the direction as a unit vector).

[5]

Question 5: Determine all critical points of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$ and classify each as corresponding to a local maximum, a local minimum, or a saddle point.

[10]

Question 6: Find the point on the plane $3x + 2y + z = 6$ that is nearest $(0, 0, 0)$. You can use whichever optimization method you like, but be sure to give some explanation of why your solution does indeed correspond to the point nearest $(0, 0, 0)$.

[10]

Question 7: Use the method of Lagrange multipliers to find the absolute maximum value of

$$f(x, y) = 2x + 2y$$

on the ellipse $x^2/16 + y^2/9 = 1$. (This problem maximizes the perimeter of a rectangle that can be inscribed in the ellipse.)

[10]