

**Question 1. [10]:**

(a) Determine the shortest distance from the point  $(3, 7, -5)$  to the  $x$ -axis.

[2]

(b) Determine if the following points all lie on the same line:  $P_1(0, -5, 5)$ ,  $P_2(1, -2, 4)$ ,  $P_3(3, 4, 2)$ .

[3]

(c) Find an equation of the largest sphere with centre  $(5, 4, 9)$  that is completely contained in the first octant. (The first octant is the region of  $\mathbb{R}^3$  with  $x \geq 0$ ,  $y \geq 0$  and  $z \geq 0$ .)

[2]

(d) Write an equation which represents the set of all points  $(x, y, z)$  which are equidistant from  $(0, 0, 2)$  and the  $xy$ -plane.

[3]

**Question 2. [10]:**

For this question use the vectors

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}, \quad \mathbf{c} = \mathbf{j} - 5\mathbf{k}$$

(a) Are vectors **a** and **b** orthogonal?

[2]

(b) Find a vector of length 4 that is orthogonal to both **a** and **b**.

[3]

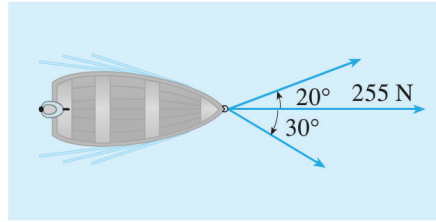
(c) Do **a**, **b** and **c** all lie in the same plane?

[2]

(d) Place the tail of each vector at  $(0,0,0)$  and consider the triangle formed by the three terminal points. What is the area of this triangle?

[3]

**Question 3. [10]:** A boat is pulled into shore using two ropes, as shown in the figure below. If a force of 255 N is needed, find the magnitude of the force in each rope.



**Question 4. [10]:**

- (a) Find the (i) vector, (ii) parametric and (iii) symmetric equations of the line through the points  $P_1(3, -1, 1)$  and  $P_2(4, 0, 2)$

[4]

- (b) Find an equation of the line through  $(1, 0, -1)$  that is parallel to the line  $\frac{x-4}{3} = \frac{y}{2} = z+2$ .

[4]

- (c) Determine the point at which the line  $x = 2 - t$ ,  $y = 1 + 3t$ ,  $z = 4t$  intersects the  $xz$ -plane.

[2]

**Question 5 [10]:**

- (a) Express the vector  $\langle 2, -1, 2 \rangle$  as the sum of a vector that is orthogonal to  $\mathbf{a} = \langle 1, 1, 1 \rangle$  and one that parallel to  $\mathbf{a}$ .

**[5]**

- (b) Find the angle between a diagonal of a cube and one of its edges.

**[5]**