Math 370 Test 1 Focus Topics

For Test 1 you should be familiar with all homework problems assigned in Asn 1, 2 & 3 as well as the theory covered up to and including Section 2.3 of the text.

As I mentioned in class, you will not be expected to produce long, original proofs of challenging, never before seen propositions. I may, however, ask you to give a short proof or two of propositions that are new to you but which I consider basic. You may also be asked to prove a result (or variation thereof) taken from the homework problems, or I may ask about some aspect of one of the proofs we worked through in class. In addition to the homework material, you should be familiar with the material outlined below.

Definitions and Concepts

Be able to

- 1. State the well ordering property of $\ensuremath{\mathbb{N}}$.
- 2. Give precise definitions of the domain and range of a function.
- 3. Define what it means for a function to be injective, surjective, bijective.
- 4. Define (in reference to sets) cardinality, finite, countably infinite, countable, uncountable.
- 5. Define (in reference to sets) bounded above/below, least upper bound, greatest lower bound.
- 6. Define ordered set, and state what it means for an ordered set to have the least upper bound property.
- 7. State what a bounded function is.
- 8. State the definition of the limit of a sequence $\lim_{n\to\infty} x_n$ and explain what it means for a sequence to converge.
- 9. State the definition of a monotone sequence.
- 10. State the definition of a subsequence.
- 11. State the definitions of $\limsup_{n\to\infty} x_n$ and $\liminf_{n\to\infty} x_n$.
- 12. For a particular sequence $\{x_n\}_{n=1}^{\infty}$, determine $\lim_{n\to\infty} x_n$ and prove your result using the ϵ , M definition.
- 13. Determine, with explanation, the lim sup and lim inf of a given sequence.

Theorems and Proofs

Know how to prove the following results:

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- 1. A countably infinite collection of countably infinite sets is countable.
- 2. The set of rational numbers is countable.
- 3. Proposition 1.2.8.
- 4. A convergent sequence has a unique limit (Prop. 2.1.6).
- 5. A convergent sequence is bounded (Prop. 2.1.7)
- 6. Proposition 2.1.10
- 7. Prop. 2.1.13. Note: not done in class, but the proof uses a variation on a technique used in class, and makes for a nice test proof.
- 8. Proposition 2.1.17
- 9. Proposition 2.2.5 (i). We did 2.2.5.(ii) in class, but (i) is easier and more direct.