Question 1. [10]:

(a) In what directions is the directional derivative of $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ at the point $P(1,1)$ equal to zero? Use unit vectors to state your answer.

[5]

(b) Determine the two points (x,y,z) on the hyperboloid $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$

[5]

Question 2. [10]:

(a) Find the absolute maximum and minimum values of $f(x, y) = x^2+y^2-2x$ on the set $D = \{(x, y) \mid x^2 + y^2 \leq 4\}$, the closed disk of radius 2 and centre $(0, 0)$.

[5]

(b) Find the point (x, y, z) on the plane $x + y + z = 1$ that is closest to the point $(2, 0, -2)$.

[5]

Question 3. [10]: Use the method of Lagrange multipliers to find the absolute maximum and minimum values of $f(x, y) = xy$ on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$. (Note: the ellipse is a closed and bounded set of points, so $f(x, y)$ will certainly have absolute extrema on the ellipse.)

[10]

Question 4. [10]:

(a) Find the volume of the region bounded above by the paraboloid $z = 16 - x^2 - y^2$ and below by the square $R : [0, 2] \times [0, 2]$ in the xy-plane.

[4]

(b) Find the volume of the region bounded above by the paraboloid $z = 16 - x^2 - y^2$ and below by the triangle in the xy-plane with vertices $(0, 0)$, $(1, 0)$ and $(0, 2)$.

[6]

Question 5 [10]:

(a) Compute $\iint_R ye^{-xy}\,dA$ where R is the rectangle $R : [0,2] \times [0,3]$.

[5]

(b) Evaluate $\int_0^2 \int_x^2 2y^2 \sin(xy)\,dy\,dx$ (reversing the order of integration may help.)

[5]