Question 1 [10 points]: Evaluate the following limits, if they exist. If a limit does not exist but is ∞ or $-\infty$, state which with an explanation of your answer.

(a) $\lim_{x\to 0} \frac{\tan(3x)}{\sin(8x)} = \lim_{x\to 0} \underbrace{\frac{\sin(3x)}{3x}}_{\to 1} \underbrace{\frac{1}{\cos(3x)}}_{\to 1} \underbrace{\frac{8x}{\sin(8x)}}_{\to 1} \underbrace{\frac{3x}{8x}}_{\to \frac{3}{8}}$

$= \boxed{\frac{3}{8}}$

[4]

(b) $\lim_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|} = \lim_{x\to 1^+} \frac{\sqrt{2x}\,\cancel{(x-1)}}{\cancel{(x-1)}}$ since $x-1 \geq 0$ as $x \to 1^+$

$= \boxed{\sqrt{2}}$

[3]

(c) $\lim_{x\to\infty} \frac{2x^3+7}{3x^3-x^2+1} = \lim_{x\to\infty} \frac{\cancel{x^3}\left(2+\frac{7}{x^3}\right)}{\cancel{x^3}\left(3-\frac{1}{x}+\frac{1}{x^3}\right)}$

$= \boxed{\frac{2}{3}}$

[3]

Question 2 [5 points]: Use the intermediate value theorem to show that the equation $\cos(x) = x$ has a solution on the interval $(0, \pi/2)$.

Let $f(x) = \cos(x) - x$, continuous on $(-\infty, \infty)$,

$f(0) = 1$, $f(\frac{\pi}{2}) = -\frac{\pi}{2}$, and since $-\frac{\pi}{2} < 0 < 1$,

$\therefore$ by the I.V.T., $f(c) = 0$ for some $0 < c < \frac{\pi}{2}$.

$\therefore$ $\cos(c) = x$.

[5]

Question 3 [5 points]: Let $f(x) = \dfrac{1}{2x-3}$. Use the definition of the derivative to find $f'(x)$. (No credit will be given if $f'(x)$ is found by any other method.)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{1}{2(x+h)-3} - \frac{1}{2x-3} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{(\cancel{2x}\cancel{-3}) - (\cancel{2x}+2h-\cancel{3})}{(2(x+h)-3)(2x-3)} \right]$$

$$= \lim_{h \to 0} \frac{1}{\cancel{h}} \frac{-2\cancel{h}}{(2(x+h)-3)(2x-3)}$$

$$= \boxed{\frac{-2}{(2x-3)^2}}$$

[5]

Question 4 [10 points]: Differentiate the following functions. It is not necessary to simplify your answers.

(a) $y = 3x^4 - 7x^3 + \pi^2$

$\boxed{y' = 12x^3 - 21x^2}$

[3]

(b) $f(x) = \frac{x^{-2}}{2} - \frac{1}{x}$ $= \frac{1}{2}x^{-2} - x^{-1}$

$\boxed{f'(x) = -x^{-3} + x^{-2}}$

[3]

(c) $y = \sin(3\pi) - \frac{\sqrt{2}}{\sqrt{x}}$ $= \sin(3\pi) - \sqrt{2}\, x^{-\frac{1}{2}}$

$\boxed{y' = \frac{\sqrt{2}}{2}\, x^{-3/2}}$

[4]

Question 5 [6 points]: Differentiate the following functions. It is not necessary to simplify your answers.

(a) $y = \dfrac{2\cos(x)}{x - \sin(x)}$

$$\boxed{y' = \frac{(x-\sin(x))(-2\sin(x)) - 2\cos(x)(1-\cos(x))}{(x-\sin(x))^2}}$$

[3]

(b) $f(x) = (x^3 - \sqrt[3]{x})\left(x - \frac{1}{x}\right)$ $= (x^3 - x^{\frac{1}{3}})(x - x^{-1})$

$$\boxed{f'(x) = (3x^2 - \tfrac{1}{3}x^{-\frac{2}{3}})(x - x^{-1}) + (x^3 - x^{\frac{1}{3}})(1 + x^{-2})}$$

[3]

Question 6 [4 points]: Find y'' if $y = \sec(x)$.

$y' = \sec(x)\tan(x)$

$y'' = \sec(x)\tan(x) \cdot \tan(x) + \sec(x)\sec^2(x)$

$= \boxed{\sec(x)\tan^2(x) + \sec^3(x)}$

[4]

Question 7 [5 points]: Find the x-coordinates of the points on the graph of $g(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 1$ at which tangent lines are parallel to the line $y = 4x - 7$.

Solve $g'(x) = 4$ for x

$$\Rightarrow \cancel{\left(\tfrac{1}{3}\right)}\cancel{3}x^2 - \cancel{\left(\tfrac{3}{2}\right)}\cancel{2}x = 4$$

$$x^2 - 3x = 4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$\boxed{x = 4, \quad x = -1}$$

[5]

Question 8 [5 points]: At time t seconds, the position function for a particle moving along a straight line is $s(t) = t^3 - 6t^2 + 9t$ metres. What is the particle's velocity when the acceleration is zero?

$$s'(t) = 3t^2 - 12t + 9$$

$$s''(t) = 6t - 12$$

$s''(t) = 0$ at $t = 2$

$$s'(2) = 3(2)^2 - 12(2) + 9 = \boxed{-3 \ \frac{m}{s}}$$

[5]