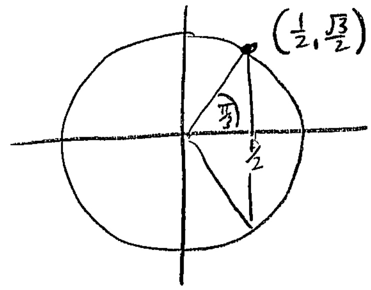
Question 1 [10 points]:

(a) Determine all values of the angle θ in $[0, 2\pi]$ such that $\sec(\theta) = 2$.

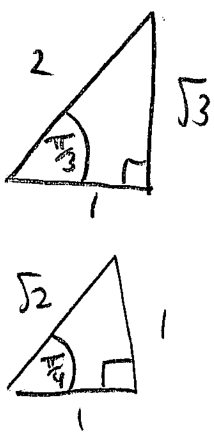
$$\sec(\theta) = 2 \Rightarrow \cos(\theta) = \frac{1}{2}$$

$$\Rightarrow \boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$$



[3]

(b) Determine the exact value and simplify: $\sin(\pi/3) - \tan(3\pi/4)$



$$= \frac{\sqrt{3}}{2} - (-1)$$

$$= 1 + \frac{\sqrt{3}}{2}$$

$$= \boxed{\frac{2+\sqrt{3}}{2}}$$

[3]

(c) Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\sin x = \tan x$.

$$\sin x = \tan x$$

$$\Rightarrow \sin x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \sin x \cos x - \sin x = 0$$

$$\Rightarrow \sin x(\cos x - 1) = 0$$

$$\Rightarrow x = 0, \pi, 2\pi \ ; \ x = 0$$

$$\boxed{\therefore x = 0, \pi, 2\pi}$$

[4]

Question 2 [3 points]: A sphere (a ball) of radius r has volume $V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$. Express the volume V as a function of the surface area S.

$$S = 4\pi r^2 \Rightarrow r = \left(\frac{S}{4\pi}\right)^{1/2}$$

$$\therefore V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{S}{4\pi}\right)^{3/2}$$

$$= \frac{4\pi S^{3/2}}{3 \cdot 4^{3/2} \cdot \pi^{3/2}}$$

$$= \frac{S^{3/2}}{3 \cdot 4^{1/2} \cdot \pi^{1/2}} = \boxed{\frac{1}{3}\sqrt{\frac{S^3}{4\pi}}}$$

[3]

Question 3 [4 points]: Let $f(x) = x^2 - 2x$. Evaluate and simplify the difference quotient $\frac{f(4+h) - f(4)}{h}$.

$$\frac{f(4+h) - f(4)}{h} = \frac{\left[(4+h)^2 - 2(4+h)\right] - \left[4^2 - 2(4)\right]}{h}$$

$$= \frac{\cancel{16} + 8h + h^2 \cancel{- 8} - 2h \cancel{- 16} \cancel{+ 8}}{h}$$

$$= \frac{h(6+h)}{h}$$

$$= \boxed{6+h}$$

[4]

Question 4 [3 points]: Determine the domain of $f(x) = \frac{\sqrt{3x-1}}{x^2-9}$.

Must have $3x - 1 \geq 0 \Rightarrow x \geq \frac{1}{3}$

and $x^2 - 9 \neq 0 \Rightarrow x \neq 3, -3$

$\boxed{\therefore \text{domain is } \left[\frac{1}{3}, 3\right) \cup (3, \infty).}$

[3]

Question 5 [3 points]: Let $H(x) = \dfrac{1}{\sqrt{3 - \cos(x)}}$. If $g(x) = 3 - x$, find functions $f(x)$ and $h(x)$ so that $H(x) = (f \circ g \circ h)(x)$.

$$\left.\begin{aligned} h(x) &= \cos(x) \\ f(x) &= \frac{1}{\sqrt{x}} \end{aligned}\right\}$$

$\underline{\text{Check:}}$ $f(g(h(x)))$.

$$= \frac{1}{\sqrt{g(h(x))}}$$

$$= \frac{1}{\sqrt{3 - h(x)}}$$

$$= \frac{1}{\sqrt{3 - \cos(x)}} = H(x)$$

[3]

Question 6 [3 points]: Let $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{1}{x^2 - 1}$. Determine $(g \circ f)(x)$ and state the domain.

$$(g \circ f)(x) = g(f(x)) = \left.\frac{1}{(\sqrt{x+1})^2 - 1}\right\} *$$

$$= \boxed{\frac{1}{x}}$$

Using $(*)$ we see that $x \neq 0$ and $x + 1 \geq 0 \Rightarrow x \geq -1$

$\therefore$ domain of $g \circ f$ is $\boxed{[-1, 0) \cup (0, \infty).}$

[3]

Question 7 [3 points]: Evaluate the following limit if it exists: $\lim\limits_{x \to 0} \dfrac{\sin^2(x)}{\cos(3x) - \sin(2x)}$

$$\lim_{x \to 0} \frac{\sin^2(x) \;\} \to 0}{\cos(3x) - \sin(2x) \;\} \to 1}$$

$$= \boxed{0}$$

[3]

Question 8 [11 points]: Evaluate the following limits, if they exist:

(a) $\lim_{x\to 1} \frac{\left(\frac{1}{x}-1\right)}{x-1}$ $\begin{matrix} \} \to \text{"}\frac{0}{0}\text{"} \\ \} \to \end{matrix}$

$= \lim_{x\to 1} \frac{(\frac{1}{x}-1)}{x-1} \cdot \frac{x}{x}$

$= \lim_{x\to 1} \frac{1-x}{x(x-1)} = \lim_{x\to 1} \frac{-\cancel{(x-1)}}{x\cancel{(x-1)}} = \boxed{-1}$

[4]

(b) $\lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h}$ $\begin{matrix} \} \to \text{"}\frac{0}{0}\text{"} \\ \} \to \end{matrix}$

$= \lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h} \cdot \frac{\sqrt{5h+4}+2}{\sqrt{5h+4}+2}$

$= \lim_{h\to 0} \frac{5\cancel{h}\ \cancel{+4}\ \cancel{-4}}{\cancel{h}(\sqrt{5h+4}+2)} = \boxed{\frac{5}{4}}$

[4]

(c) $\lim_{x\to 2} \frac{x^2+5x+6}{\sqrt{x+2}-2}$ $\begin{matrix} \} \to \text{"}\frac{20}{0}\text{"} \\ \} \to \end{matrix}$

$\boxed{\therefore \text{ limit does not exist.}}$

[3]

Question 9 [10 points]: Evaluate the following limits, if they exist:

(a) $\lim_{t\to -1} \frac{t^2 - t - 2}{t^2 + 3t + 2}$ $\begin{matrix} \} \to \\ \} \to \end{matrix}$ "$\frac{0}{0}$"

$$= \lim_{t\to -1} \frac{\cancel{(t+1)}(t-2)}{\cancel{(t+1)}(t+2)}$$

$$= \boxed{-3}$$

[5]

(b) $\lim_{x\to 0^+} \left(\frac{1}{x^2+x}\right) - \frac{1}{x}$ $\to$ "$\infty - \infty$"

$$= \lim_{x\to 0^+} \frac{1-(x+1)}{x(x+1)}$$

$$= \lim_{x\to 0^+} \frac{-\cancel{x}}{\cancel{x}(x+1)}$$

$$= \boxed{-1}$$

[5]