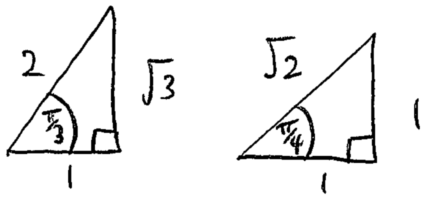
Question 1 [10 points]:

(a) Determine the exact value and simplify: $\cos(\pi/3) - \tan(3\pi/4)$



$$= \frac{1}{2} - \left(\frac{-1}{1}\right)$$

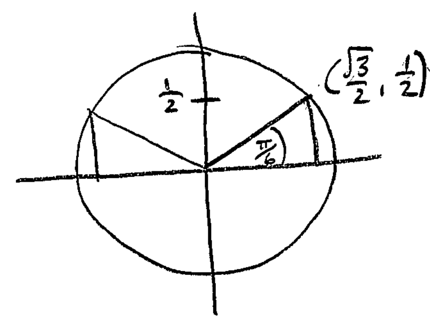
$$= \boxed{\frac{3}{2}}$$

[3]

(b) Determine all values of the angle θ in $[0, 2\pi]$ such that $\csc(\theta) = 2$.

$$\csc(\theta) = 2 \Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}}$$



[3]

(c) Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\sin x = \tan x$.

$$\sin x = \tan x$$

$$\Rightarrow \sin x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \sin x \cos x - \sin x = 0$$

$$\Rightarrow \sin x (\cos x - 1) = 0$$

$$\Rightarrow \sin x = 0; \quad \cos x = 1$$

$$\Rightarrow x = 0, \pi, 2\pi; \quad x = 0$$

$$\boxed{\therefore\ x = 0, \pi, 2\pi}$$

[4]

Question 2 [3 points]: Determine the domain of $f(x) = \dfrac{\sqrt{2x-1}}{x^2-9}$.

Must have $2x-1 \geq 0 \Rightarrow x \geq \frac{1}{2}$

and $x^2-9 \neq 0 \Rightarrow x \neq 3, -3$.

$\therefore$ domain is $\boxed{[\tfrac{1}{2}, 3) \cup (3, \infty)}$.

[3]

Question 3 [4 points]: Let $f(x) = x^2 - 2x$. Evaluate and simplify the difference quotient $\dfrac{f(3+h)-f(3)}{h}$.

$$\frac{f(3+h)-f(3)}{h} = \frac{[(3+h)^2 - 2(3+h)] - [3^2 - 2(3)]}{h}$$

$$= \frac{\cancel{9} + 6h + h^2 - \cancel{6} - 2h - \cancel{9} + \cancel{6}}{h}$$

$$= \frac{h(h+4)}{h}$$

$$= \boxed{h+4}$$

[4]

Question 4 [3 points]: A sphere (a ball) of radius r has volume $V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$. Express the surface area S as a function of the volume V.

$$V = \frac{4}{3}\pi r^3 \Rightarrow r = \left(\frac{3V}{4\pi}\right)^{1/3}$$

$$\therefore S = 4\pi r^2 = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3} = \frac{4\pi \cdot 3^{2/3} \cdot V^{2/3}}{4^{2/3}\pi^{2/3}}$$

$$= 4^{1/3}\pi^{1/3}3^{2/3}V^{2/3}$$

$$= \boxed{(36\pi V^2)^{1/3}}$$

[3]

Question 5 [3 points]: Let $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x^2-1}$. Determine $(g \circ f)(x)$ and state the domain.

$$(g \circ f)(x) = g(f(x)) = \left.\frac{1}{(\sqrt{x+1})^2 - 1}\right\} (*)$$

$$= \boxed{\frac{1}{x}}$$

Using $(*)$ we see that $x \neq 0$, and $x + 1 \geq 0 \Rightarrow x \geq -1$

$\therefore$ domain of $g \circ f$ is $\boxed{[-1, 0) \cup (0, \infty)}$

[3]

Question 6 [3 points]: Let $H(x) = \frac{1}{\sqrt{\sin(x) + 2}}$. If $g(x) = x + 2$, find functions $f(x)$ and $h(x)$ so that $H(x) = (f \circ g \circ h)(x)$.

$h(x) = \sin(x)$

$f(x) = \frac{1}{\sqrt{x}}$

Check: $f(g(h(x)))$

$$= \frac{1}{\sqrt{g(h(x))}}$$

$$= \frac{1}{\sqrt{h(x) + 2}}$$

$$= \frac{1}{\sqrt{\sin(x) + 2}} = H(x)$$

[3]

Question 7 [4 points]: Evaluate the following limit if it exists: $\lim_{x \to 0} \frac{\cos(3x) - \sin(2x)}{\sin^2(x)}$

$$\lim_{x \to 0} \frac{\cos(3x) - \sin(2x)}{\sin^2(x)} \quad \begin{matrix} \} \to \text{"} 1 \\ \} \to 0 \text{"} \end{matrix}$$

$\therefore$ Limit does not exist.

[3]

Question 8 [10 points]: Evaluate the following limits, if they exist:

(a) $\lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h}$ $\begin{matrix} \} \to \\ \} \to \end{matrix}$ "$\frac{0}{0}$"

$$= \lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h} \cdot \frac{\sqrt{5h+4}+2}{\sqrt{5h+4}+2}$$

$$= \lim_{h\to 0} \frac{5h\cancel{+4}\cancel{-4}}{h(\sqrt{5h+4}+2)}$$

$$= \lim_{h\to 0} \frac{5}{\sqrt{5h+4}+2} = \boxed{\frac{5}{4}}$$

[4]

(b) $\lim_{x\to 1} \frac{\left(\frac{1}{x}-1\right)}{x-1}$ $\begin{matrix} \} \to \\ \} \to \end{matrix}$ "$\frac{0}{0}$"

$$= \lim_{x\to 1} \frac{\left(\frac{1}{x}-1\right)}{x-1} \cdot \frac{x}{x}$$

$$= \lim_{x\to 1} \frac{1-x}{x(x-1)} = \lim_{x\to 1} \frac{-\cancel{(x-1)}}{x\cancel{(x-1)}} = \boxed{-1}$$

[4]

(c) $\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x^2+5x+6}$ $\begin{matrix} \} \to \\ \} \to \end{matrix}$ $\frac{0}{20}$

$$= \boxed{0}$$

[2]

Question 9 [10 points]: Evaluate the following limits, if they exist:

(a) $\lim_{t\to-1} \frac{t^2+3t+2}{t^2-t-2}$ $\begin{matrix} \} \to \\ \} \to \end{matrix}$ "$\frac{0}{0}$"

$$= \lim_{t\to-1} \frac{\cancel{(t+1)}(t+2)}{\cancel{(t+1)}(t-2)}$$

$$= \frac{1}{-3}$$

$$= \boxed{\frac{-1}{3}}$$

[5]

(b) $\lim_{x\to0^+} \left(\frac{1}{x} - \frac{1}{x^2+x}\right)$ $\longrightarrow$ "$\infty - \infty$"

$$= \lim_{x\to0^+} \frac{x\cancel{+1}\cancel{-1}}{x(x+1)}$$

$$= \lim_{x\to0^+} \frac{1}{x+1}$$

$$= \boxed{1}$$

[5]