Question 1 [10 points]:

(a) Determine the exact value and simplify: $\cos(\pi/3) - \tan(3\pi/4)$

[3]

(b) Determine all values of the angle θ in $[0, 2\pi]$ such that $\csc(\theta) = 2$.

[3]

(c) Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\sin x = \tan x$.

[4]

Question 2 [3 points]: Determine the domain of $f(x) = \dfrac{\sqrt{2x-1}}{x^2 - 9}$.

[3]

Question 3 [4 points]: Let $f(x) = x^2 - 2x$. Evaluate and simplify the difference quotient $\dfrac{f(3+h) - f(3)}{h}$.

[4]

Question 4 [3 points]: A sphere (a ball) of radius r has volume $V = \dfrac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$. Express the surface area S as a function of the volume V.

[3]

Question 5 [3 points]: Let $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{1}{x^2 - 1}$. Determine $(g \circ f)(x)$ and state the domain.

[3]

Question 6 [3 points]: Let $H(x) = \dfrac{1}{\sqrt{\sin(x) + 2}}$. If $g(x) = x + 2$, find functions $f(x)$ and $h(x)$ so that $H(x) = (f \circ g \circ h)(x)$.

[3]

Question 7 [3 points]: Evaluate the following limit if it exists: $\displaystyle \lim_{x \to 0} \dfrac{\cos(3x) - \sin(2x)}{\sin^2(x)}$

[3]

Question 8 [11 points]: Evaluate the following limits, if they exist:

(a) $\displaystyle\lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h}$

[4]

(b) $\displaystyle\lim_{x\to 1} \frac{\left(\dfrac{1}{x}-1\right)}{x-1}$

[4]

(c) $\displaystyle\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x^2+5x+6}$

[3]

Question 9 [10 points]: Evaluate the following limits, if they exist:

(a) $\displaystyle\lim_{t \to -1} \frac{t^2 + 3t + 2}{t^2 - t - 2}$

[5]

(b) $\displaystyle\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$

[5]

$\displaystyle\lim_{t \to -1} \frac{t^2 + 3t + 2}{t^2 - t - 2}$