

### 3 Curve Sketching

So far we have seen that

- (i) If  $f'(x) > 0$  on an interval then the graph of  $y = f(x)$  is increasing on the interval;
- (ii) If  $f'(x) < 0$  on an interval then the graph of  $y = f(x)$  is decreasing on the interval;
- (iii) If  $f''(x) > 0$  on an interval then the graph of  $y = f(x)$  is concave up on the interval;
- (iv) If  $f''(x) < 0$  on an interval then the graph of  $y = f(x)$  is concave down on the interval.

Using this information we then located relative extrema and inflection points, and we sketched a fairly accurate picture of the graph of  $y = f(x)$ .

We now improve our graph by making use of additional information:

- (i) The  $x$ -intercepts of  $y = f(x)$ ,
- (ii) the  $y$ -intercept of  $y = f(x)$ ,
- (iii) the horizontal asymptotes of  $y = f(x)$ , and
- (iv) the vertical asymptotes of  $y = f(x)$ .

#### Example 1

Let  $f(x) = \frac{(x-1)^2}{(x-3)^2}$ . Determine the

- (i)  $x$ -intercepts
- (ii)  $y$ -intercepts
- (iii) vertical asymptotes
- (iv) horizontal asymptotes
- (v) intervals of increase/decrease
- (vi) local extreme values
- (vii) intervals of concavity
- (viii) inflection points

Sketch the graph of  $y = f(x)$ .

**Example 2**

Suppose we have analyzed the function  $y = f(x)$  and found the following information:

(i) The domain of  $f$  is  $(-\infty, 1) \cup (1, \infty)$ .

(ii)  $f(x)$  has the following function values:

$x$	-3	-2	-1	-1/2	0	1/2	3	4
$f(x)$	3/2	2	1	0	-1/2	0	-1	-3/2

(iii)  $\lim_{x \rightarrow -\infty} f(x) = 1$ ,  $\lim_{x \rightarrow \infty} f(x) = -2$

(iv)  $\lim_{x \rightarrow 1^-} f(x) = \infty$ ,  $\lim_{x \rightarrow 1^+} f(x) = -\infty$

(v)  $f'(-2) = f'(0) = f'(3) = 0$

(vi)  $f'(x) > 0$  on  $(-\infty, -2)$ ,  $(0, 1)$  and  $(1, 3)$ ;  
 $f'(x) < 0$  on  $(-2, 0)$  and  $(3, \infty)$

(vii)  $f''(-3) = f''(-1) = f''(4) = 0$

(viii)  $f''(x) > 0$  on  $(-\infty, -3)$ ,  $(-1, 1)$  and  $(4, \infty)$ ;  
 $f''(x) < 0$  on  $(-3, -1)$  and  $(1, 4)$

Sketch the graph of  $y = f(x)$ .