

1. Simplify and express in the form $a + bi$: $\frac{5 + 5i}{(1 + 3i)(\frac{1}{2} - \frac{i}{2})}$.
2. Determine and sketch all cube roots of $8(1 - \sqrt{3}i)$.
3. Evaluate $|e^{iz}|$ if $z = 6e^{i\pi/3}$.
4. Determine the points, if any, at which $f(z) = |\bar{z} - i|^2$ is analytic.
5. Determine the harmonic conjugate of $u(x, y) = 3x^2y - y^3 + x + 4xy$.
6. Can $u(x, y) = xy^2$ be the real part of an entire function? Explain.
7. Find all values of $i^{\text{Log}(i)}$.
8. Evaluate $\int_{\Gamma} \text{Re}(z) dz$ where
 - (a) Γ is a line from $z = 0$ to $z = 1 + i$
 - (b) Γ is a line segment from $z = 0$ to $z = i$ followed by a line segment from $z = i$ to $z = 1 + i$.
9. Evaluate $\int_C \frac{\cos z}{e^z - 1} dz$ where C is the circle $|z - 2i| = 1$ traversed once in the positive direction.
10. Evaluate $\int_{\Gamma} \frac{1}{z} dz$ where Γ is any simple contour from $z = -2$ to $z = -i$ which does not leave the third quadrant.
11. Evaluate $\int_C \frac{z^3}{(z + i)(z + 2)^2} dz$ where C is the circle
 - (a) $|z| = 1/2$
 - (b) $|z| = 3/2$
 - (c) $|z + 2| = 1/2$
 - (d) $|z| = 3$In each case the circle is traversed once in the positive direction.
12. Expand $f(z) = \frac{1}{(z + 1)(z + 3)}$ in a Laurent series valid for $1 < |z| < 3$.
13. Show that $f(z) = \frac{1 + \cos(\pi z)}{(z^2 - 1)^2}$ has a removable singularity at $z = -1$ (You may use L'Hospital's rule here.)
14. Use the residue theorem to evaluate the following integrals. In each case the circles are traversed once in the positive direction:
 - (a) $\int_{|z|=2} \frac{z^3 + 2z}{z - i} dz$
 - (b) $\int_{|z|=1} z^2 e^{1/z} dz$