1. Evaluate the following integrals over the indicated contours. In each case the contour is traversed once in the positive direction. State any theorems used in your calculation.

   (a) $\displaystyle\int_\Gamma \frac{z^2}{z+3}\,dz$ where Γ is the circle of radius 2 and centre $z = 0$.

   (b) $\displaystyle\int_\Gamma \frac{z^2}{z+3}\,dz$ where Γ is the circle of radius 4 and centre $z = 0$.

   (c) $\displaystyle\int_\Gamma \frac{2\cos^2 z}{z^2+7z+7}\,dz$ where Γ is the unit circle .

   (d) $\displaystyle\int_\Gamma \frac{e^z}{z}\,dz$ where Γ is the square bounded by the lines $|\mathrm{Re}(z)| = 2$ and $|\mathrm{Im}(z)| = 2$.

   (e) $\displaystyle\int_\Gamma \frac{z^2}{(z^2+1)^2}\,dz$ where Γ is the semi-circle $\{z : z = t + 0i, -R \le t \le R\} \cup \{z : z = Re^{it}, 0 \le t \le \pi\}$.

2. Prove the *Maximum Modulus Principle*: If $f(z)$ is analytic inside and on a simple closed contour Γ then the maximum of $|f(z)|$ occurs on Γ.

   Hint: proceed as follows: Let z_0 be any point inside Γ and let M be the maximum of $|f(z)|$ on Γ. We will show that $|f(z_0)| \le M$.

   (a) Let $n \ge 1$ be an integer. Then

   $$[f(z_0)]^n = \frac{1}{2\pi i}\int_\Gamma \frac{[f(\zeta)]^n}{\zeta - z_0}d\zeta \quad \text{(why?)} \tag{1}$$

   (b) Let μ be the minimum distance from z_0 to Γ and $\ell(\Gamma)$ be the length of Γ. Use (1) to show that

   $$|f(z_0)|^n \le \frac{1}{2\pi}\frac{M^n}{\mu}\ell(\Gamma) \tag{2}$$

   (c) Take n^{th} roots of both sides of (2) and then let $n \to \infty$.