Question 1. [10]:

(a)[5] Compute $\iint_D (x^2 + 2y)\, dA$ where D is the region in the first quadrant that is bounded between the curves $y = x$ and $y = x^3$.

(b)[5] Evaluate $\int_0^1 \int_x^1 e^{x/y}\, dy\, dx$ by first changing the order of integration.

Question 2. [10]: The upper hemisphere of a sphere of radius a is given by $z = \sqrt{a^2 - x^2 - y^2}$. Determine the volume of the hemisphere using polar coordinates.
(You may know the answer directly thanks to a well-known formula; you are required here to show how to get that answer.)

Question 3. [10]: Evaluate $\iiint\limits_E y\,dA$ where E is the solid region in the first octant that lies under the plane $3x + y + z = 1$.

Question 4. [10]: The sphere B of radius a has equation $x^2 + y^2 + z^2 = a^2$ and volume $V(B) = 4\pi a^3/3$. The average distance from a point in the sphere to its centre is defined to be

$$\rho_{\text{avg}} = \frac{1}{V(B)} \iiint\limits_B \sqrt{x^2 + y^2 + z^2}\, dV$$

Compute ρ_{avg} .

Question 5 [10]: For this question we will use a transformation to evaluate $\iint\limits_R (x+y)e^{x^2-y^2}\,dA$ where R is the rectangle with vertices $(0,0)$, $(3/2,3/2)$, $(1,-1)$, $(5/2,1/2)$.

(a)[2] The transformation is $u = x+y,\ v = x-y$. Determine $\dfrac{\partial(x,y)}{\partial(u,v)}$, the Jacobian of the transformation.

(b)[3] Determine the region in the uv-plane that maps onto R under the transformation.

(c)[5] Use parts (a) and (b) to evaluate $\iint\limits_R (x+y)e^{x^2-y^2}\,dA$.