**Question 1. [10]:**

**(a)[5]** Determine the direction derivative of $f(x, y, z) = x^2y + x\sqrt{1+z}$ at the point $(1, 2, 3)$ in the direction of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ .

**(b)[5]** Determine an equation of the tangent plane to the surface

$$\sin(xyz) = x + 2y + 3z$$

at the point $(2, -1, 0)$ .

**Question 2. [10]:** Determine all critical points of $f(x, y) = x(x^2 + xy + y^2 - 9)$ and classify each as a corresponding to a local maximum, a local minimum, or a saddle point.

**Question 3. [10]:** Find three positive numbers whose sum is 100 and whose product is a maximum. You may use any method you like, but be sure to justify that your three numbers do indeed correspond to the absolute maximum.

**Question 4. [10]:** Use the method of Lagrange multipliers to find the absolute maximum value of

$$f(x, y) = 2x^2 + 3y^2 - 4x - 5$$

on the circle $x^2 + y^2 = 16$ .

**Question 5 [10]:**

**(a)[5]** Evaluate

$$\int_1^2 \int_0^1 (x+y)^{-2}\, dx\, dy$$

**(b)[5]** Determine the volume of the solid in the first octant which is enclosed by the surface $z = 1 + e^x \sin(y)$ and the planes $x = 1$, $y = \pi$ .