Question 1: Determine the following limits. If a limit does not exist because it is ∞ or $-\infty$, state which with an explanation of your reasoning.

(a)[3] $\displaystyle\lim_{x\to -3^+} \frac{-2}{3+x}$ $\rightarrow$ "$\dfrac{-2}{0^+}$"

$= \boxed{-\infty}$

(b)[3] $\displaystyle\lim_{x\to\infty} \frac{3-2x-2x^3}{7+x^2-5x^3}$

$$= \lim_{x\to\infty} \frac{\cancel{x^3}\left(\frac{3}{x^3} - \frac{2}{x^2} - 2\right)}{\cancel{x^3}\left(\frac{7}{x^3} + \frac{1}{x} - 5\right)}$$

$= \boxed{\dfrac{2}{5}}$

(b)[4] $\displaystyle\lim_{x\to 4^-} \frac{x+4}{x^2-16}$ $\rightarrow \dfrac{8}{0^-}$

$= \boxed{-\infty}$

Question 2: Determine the derivative of each of the following functions. (It is not necessary to simplify your final answers):

(a)[3] $y = (x^3 + 7x^2)(x^3 - x^2 + 5)$

$$y' = (3x^2+14x)(x^3-x^2+5) + (x^3+7x^2)(3x^2-2x)$$

(b)[3] $f(x) = x\sqrt{x} + \frac{x^2}{2} = x^{3/2} + \frac{1}{2}x^2$

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + x$$

(c)[4] $g(x) = \frac{x^2+6}{\sqrt{x^2+5}} = \frac{x^2+6}{(x^2+5)^{1/2}}$

$$g'(x) = \frac{(x^2+5)^{1/2}(2x) - (x^2+6)\,\frac{1}{2}(x^2+5)^{-\frac{1}{2}}(2x)}{((x^2+5)^{1/2})^2}$$

Question 3: Determine the derivative of each of the following functions. (It is not necessary to simplify your final answers):

(a)[3] $y = \frac{3}{(3x^2 - x)^{2/3}}$ $= 3(3x^2 - x)^{-2/3}$

$y' = (3)(-\frac{2}{3})(3x^2 - x)^{-5/3}(6x-1)$

$= -2(3x^2 - x)^{-5/3}(6x-1)$

(b)[3] $f(x) = (3x^4)e^{-x}$

$f'(x) = 12x^3 e^{-x} + 3x^4 e^{-x}(-1)$

$= 12x^3 e^{-x} - 3x^4 e^{-x}$

(c)[4] $g(x) = \frac{(x + e^{2x})^2}{3x}$

$$g'(x) = \frac{3x \cdot 2(x+e^{2x})(1+2e^{2x}) - (x+e^{2x})^2(3)}{(3x)^2}$$

$$= \frac{2x(x+e^{2x})(1+2e^{2x}) - (x+e^{2x})^2}{3x^2}$$

Question 4:

(a)[5] Determine an equation of the tangent line to the graph of $y = (x+3)^3$ at the point where $x = -1$.

At $x = -1$, $y = (-1+3)^3 = 8$

$\frac{dy}{dx} = 3(x+3)^2$; $\left.\frac{dy}{dx}\right|_{x=-1} = 3(-1+3)^2 = 12$

$\therefore$ Equation of tangent line is

$$\boxed{y - 8 = 12(x+1)}$$

(b)[5] Suppose m employees produce $q = \frac{200m - m^2}{20}$ units of product per day, and that the demand equation for the product is $p = -0.1q + 70$ where p is price in dollars and q is the number of units. Determine the marginal revenue product when $m = 40$. State units with your answer.

$r = pq = [-0.1q + 70]\,q = -0.1q^2 + 70q.$

$\frac{dr}{dm} = \frac{dr}{dq} \cdot \frac{dq}{dm} = (-0.2q + 70)\left[\frac{1}{20}(200 - 2m)\right].$

When $m = 40$, $q = \frac{(200)(40) - 40^2}{20} = 320$,

So $\left.\frac{dr}{dm}\right|_{m=40} = \left(\left.\frac{dr}{dq}\right|_{q=320}\right)\left(\left.\frac{dq}{dm}\right|_{m=40}\right)$

$= [(-0.2)(320) + 70]\left[\frac{1}{20}(200 - 2 \cdot 40)\right]$

$= 36$ \$/day per employee.

Question 5:

(a)[5] Suppose the average cost of producing q units of a product is

$$\bar{c} = \frac{850}{q} + 4000\frac{e^{(2q+6)/800}}{q}$$

Determine the marginal cost when $q = 97$.

$$c = q\bar{c} = 850 + 4000\, e^{\frac{2q+6}{800}}$$

$$\frac{dc}{dq} = 4000 e^{\frac{2q+6}{800}} \cdot \left(\frac{2}{800}\right) = 10 e^{\frac{2q+6}{800}}$$

$$\therefore \left.\frac{dc}{dq}\right|_{q=97} = 10\, e^{\frac{2(97+6)}{800}} \approx \boxed{12.94 \text{ \$/unit.}}$$

(b)[5] The demand equation for a product is $p = -0.1q + 500$. At what production level q is marginal revenue zero?

$$r = pq = -0.1q^2 + 500q.$$

Solve $\frac{dr}{dq} = 0$ for q.

$$\frac{dr}{dq} = -0.2q + 500.$$

$$\frac{dr}{dq} = 0 \Rightarrow -0.2q + 500 = 0$$

$$\Rightarrow q = \frac{500}{0.2} = \boxed{2500 \text{ units.}}$$