Question 1:

(a)[5] Solve for t:

$$\frac{2t+1}{2t+3} = \frac{3t-1}{3t+4}$$

$$(2t+1)(3t+4) = (3t-1)(2t+3)$$

$$6t^2+11t+4 = 6t^2+7t-3$$

$$4t = -7$$

$$\boxed{t = \frac{-7}{4}}$$

(b)[5] Solve for x:

$$x + \sqrt{4x} - 5 = 0$$

$$(\sqrt{4x})^2 = (5-x)^2$$

$$4x = 25 - 10x + x^2$$

$$x^2 - 14x + 25 = 0$$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{14 \pm 4\sqrt{6}}{2}$$

$$= 7 \pm 2\sqrt{6}$$

BUT...

Check:

$x = 7+2\sqrt{6}$:

$(7+2\sqrt{6}) + \sqrt{4(7+2\sqrt{6})} - 5 \neq 0$

$x = 7-2\sqrt{6}$:

$(7-2\sqrt{6}) + \sqrt{4(7-2\sqrt{6})} - 5 = 0$

$\therefore x = 7 - 2\sqrt{6}$ is the only solution

Question 2:

(a)[3] Let $h(x) = \frac{3}{x^2+x+1}$ and $g(x) = x^2 + x$. Find a function $f(x)$ so that $h(x) = (f \circ g)(x)$.

$$h(x) = \frac{3}{x^2+x+1} = \frac{3}{g(x)+1}$$

$$\therefore \; f(x) = \frac{3}{x+1}$$

Check: $f(g(x)) = \frac{3}{g(x)+1} = \frac{3}{x^2+x+1} = h(x).$

(b)[4] Determine the domain of $p(x) = \frac{x+5}{x^2+x-6}$.

Require $x^2+x-6 \neq 0$

$\Rightarrow (x-2)(x+3) \neq 0$

$\therefore \; x \neq 2, \; x \neq -3.$

$\therefore$ domain is $(-\infty,-3)\cup(-3,2)\cup(2,\infty).$

(c)[3] Let $q(x) = \frac{4}{3}\pi x^3$. Determine the inverse, $q^{-1}(x)$.

$$y = \frac{4}{3}\pi x^3$$

$$x = \left(\frac{3y}{4\pi}\right)^{1/3}$$

$x \leftrightarrow y$: $y = \left(\frac{3x}{4\pi}\right)^{1/3}$

$$\therefore \; q^{-1}(x) = \left(\frac{3x}{4\pi}\right)^{1/3}.$$

Question 3 [10]: The weekly demand for a company's product is $q = 13$ units when the price is $p = \$24$ per unit, while a price of $p = \$48$ per unit lowers demand to $q = 11$.

(a)[5] Determine the demand equation relating p and q, assuming it is linear.

p	q
24	13
48	11

$$m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{13-11}{24-48} = \frac{2}{-24} = \frac{-1}{12}$$

$$\therefore \quad q - 13 = \frac{-1}{12}(p - 24)$$

$$\underline{\underline{\text{or}}} \quad q = -\frac{1}{12}p + 15$$

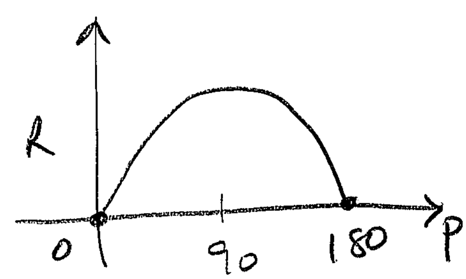
$$\underline{\underline{\text{or}}} \quad p = 180 - 12q$$

(b)[5] What price p will maximize total weekly revenue?

$$R = pq = p\left(\frac{-1}{12}p + 15\right)$$

$$R = 0 \Rightarrow p = 0 \quad \text{or} \quad p = (15)(12) = 180$$

$\therefore$ Revenue function has graph



$\therefore$ Revenue is maximized at $p = \$90.$

Question 4: Evaluate the following limits, if they exist:

(a)[4] $\lim_{t\to 0} \frac{t^3 - 3t^2}{t^3 - 4t^2}$ ~ 0 / ~ 0

$$= \lim_{t\to 0} \frac{\cancel{t^2}(t-3)}{\cancel{t^2}(t-4)}$$

$$= \boxed{\frac{3}{4}}$$

(b)[3] $\lim_{x\to 3} \frac{x^2 - 2x - 3}{x^2 - 2x + 15}$ $\left.\right\} \to \frac{0}{18}$

$$= 0$$

(c)[3] $\lim_{x\to -2} \frac{x^2 - 3x - 10}{x^2 + 5x + 6}$ $\to 0$ / $\to 0$

$$= \lim_{x\to -2} \frac{\cancel{(x+2)}(x-5)}{\cancel{(x+2)}(x+3)}$$

$$= \frac{-7}{1}$$

$$= \boxed{-7}$$

Question 5:

(a)[5] Evaluate the following limit, if it exists: $\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6}$

$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} \cdot \frac{\sqrt{x-2}+2}{\sqrt{x-2}+2}$$

$$= \lim_{x\to 6} \frac{x-2-4}{(x-6)(\sqrt{x-2}+2)}$$

$$= \lim_{x\to 6} \frac{\cancel{(x-6)}}{\cancel{(x-6)}(\sqrt{x-2}+2)}$$

$$= \boxed{\frac{1}{4}}$$

(b)[5] Let $f(x) = x^2 - 5$. Evaluate

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h)^2-5]-[x^2-5]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2hx+h^2\cancel{-5}\cancel{-x^2}\cancel{+5}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \boxed{2x}.$$