 Name: SOLUTIONS

(1) [5] Determine dy/dx by implicit differentiation:

$$\ln(xy) = e^{xy}$$

$$\frac{d}{dx}[\ln(xy)] = \frac{d}{dx}[e^{xy}]$$

$$\frac{1}{xy}[y + xy'] = e^{xy}[y + xy']$$

$$\frac{1}{x} + \frac{1}{y}y' = ye^{xy} + xe^{xy}y'$$

$$y'\left[\frac{1}{y} - xe^{xy}\right] = ye^{xy} - \frac{1}{x}$$

$$\boxed{\therefore\ y' = \frac{ye^{xy} - \frac{1}{x}}{\frac{1}{y} - xe^{xy}}}$$

(2) [5] Determine y' if $y = x^{\sqrt{x}}$. (Logarithmic differentiation may be helpful here.)

$$\ln(y) = x^{\frac{1}{2}} \ln x$$

$$\frac{1}{y}y' = \frac{1}{2}x^{-\frac{1}{2}}\ln x + x^{\frac{1}{2}}\frac{1}{x}$$

$$\boxed{y' = x^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln x + \frac{1}{\sqrt{x}}\right]}$$

(3) [5] Suppose you are using Newton's method to find a root of $x^4 = 3x - 1$ that is between $x = 0$ and $x = 1$. Using a starting value of $x_1 = 0.5$, determine x_2, the next approximation given by Newton's method.

$$x^4 - 3x + 1 = 0$$

$$f(x) = x^4 - 3x + 1$$

$$f'(x) = 4x^3 - 3$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.5 - \frac{(0.5)^4 - 3(0.5) + 1}{4(0.5)^3 - 3}$$

$$= \boxed{0.325}$$