

(1) [7] Let $f(x) = \frac{1}{x-1}$. Find and simplify

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{\left(\frac{1}{x+h-1} - \frac{1}{x-1} \right)}{h}$$

$$= \frac{1}{h} \left[\frac{\cancel{x-1} - \cancel{x-h+1}}{(x+h-1)(x-1)} \right]$$

$$= \frac{1}{h} \left[\frac{-h}{(x+h-1)(x-1)} \right]$$

$$= \frac{-1}{(x+h-1)(x-1)}$$

(2) [4] Let $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x+2}$. Find and simplify $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x))$$

$$= \frac{1}{(g(x))^2 + 1}$$

$$= \frac{1}{(\sqrt{x+2})^2 + 1}$$

$$= \frac{1}{x+2+1}$$

$$= \boxed{\frac{1}{x+3}}$$

(3) [4] Let $h(x) = \sqrt{\frac{x^2-1}{x+3}}$. Find functions f and g so that $h(x) = (f \circ g)(x)$.

$$\text{Let } g(x) = \frac{x^2-1}{x+3}$$

$$f(x) = \sqrt{x}$$

$$\text{Then } (f \circ g)(x) = f(g(x))$$

$$= \sqrt{\frac{x^2-1}{x+3}}$$

$$= h(x).$$