Question 1:

(a)[2] Determine $\frac{d}{dx}\left[\underbrace{\int_0^1 e^{\arctan(x)}\,dx}_{\text{constant}}\right]$. $= \boxed{0}$

(b)[2] Determine $\int_0^1 \frac{d}{dx}[e^{\arctan(x)}]\,dx$.

$$= \left[e^{\arctan(x)}\right]_0^1$$

$$= e^{\pi/4} - e^0 = \boxed{e^{\frac{\pi}{4}} - 1}$$

(c)[2] Determine $\frac{d}{dx}\left[\int_0^x e^{\arctan(t)}\,dt\right]$. $= \boxed{e^{\arctan(x)}}$ by F.T.C.

(d)[4] Determine $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$. let $u = x^{\frac{1}{2}}$, $du = \frac{1}{2}x^{-\frac{1}{2}}\,dx$

$$\therefore I = 2\int \sin(u)\,du$$

$$= -2\cos(u) + C$$

$$= \boxed{-2\cos(\sqrt{x}) + C}$$

Question 2:

(a)[4] Determine $\int (t-1)\ln(t)\,dt = I$

$$\left.\begin{matrix} u = \ln(t) \; ; \; dv = (t-1)\,dt \\ du = \frac{1}{t}dt \; ; \; v = \frac{t^2}{2} - t \end{matrix}\right\}$$

$$\therefore I = \int u\,dv = uv - \int v\,du$$

$$= \ln(t)\cdot\left(\frac{t^2}{2} - t\right) - \int\left(\frac{t^2}{2} - t\right)\left(\frac{1}{t}\right)dt$$

$$= \ln(t)\cdot\left(\frac{t^2}{2} - t\right) - \int\left(\frac{t}{2} - 1\right)dt$$

$$= \boxed{\ln(t)\cdot\left(\frac{t^2}{2} - t\right) - \frac{t^2}{4} + t + C}$$

(b)[6] Determine $\int e^{-x}\cos(x)\,dx = I$

$$u = e^{-x} \; ; \; dv = \cos(x)\,dx$$
$$du = -e^{-x}dx \; ; \; v = \sin(x)$$

$$\therefore I = \int u\,dv = uv - \int v\,du$$

$$= e^{-x}\sin(x) + \underbrace{\int \sin(x)(e^{-x})\,dx}_{\begin{matrix} u = e^{-x} & dv = \sin(x)dx \\ du = -e^{-x}dx & v = -\cos(x) \end{matrix}}$$

$$= e^{-x}\sin(x) + \left[e^{-x}(-\cos(x)) - \underbrace{\int e^{-x}\cos(x)\,dx}_{I}\right]$$

$$\therefore I = e^{-x}\sin(x) - e^{-x}\cos(x) - I$$

$$\boxed{\therefore I = \frac{e^{-x}(\sin(x) - \cos(x))}{2} + C}$$

Question 3:

(a)[7] Evaluate $\int_0^{\pi/2} \sin^3(x)\cos^3(x)\,dx = \int_0^{\frac{\pi}{2}} \sin^3(x)\,(1-\sin^2(x))\cos(x)\,dx$

let $u = \sin(x)$, $du = \cos(x)\,dx$ $\Big\}$ $x=0 \Rightarrow u=\sin(0)=0$, $x=\frac{\pi}{2} \Rightarrow u=\sin(\frac{\pi}{2})=1$

$$\therefore I = \int_0^1 u^3(1-u^2)\,du$$

$$= \left[\frac{u^4}{4} - \frac{u^6}{6}\right]_0^1$$

$$= \frac{1}{4} - \frac{1}{6}$$

$$= \boxed{\frac{1}{12}}$$

(b)[3] Determine $\int \cot(\pi x)\,dx = \int \frac{\cos(\pi x)}{\sin(\pi x)}\,dx = I$

let $u = \sin(\pi x)$

$du = \cos(\pi x)\,\pi\,dx$

$$\therefore I = \frac{1}{\pi}\int \frac{1}{u}\,du$$

$$= \frac{1}{\pi}\ln|u| + C = \boxed{\frac{1}{\pi}\ln|\sin(\pi x)| + C}$$

Question 4 [10 points]: Determine $\int \frac{128}{x^3\sqrt{x^2-16}}\,dx = I$

Let $x = 4\sec\theta \Rightarrow \sec\theta = \frac{x}{4}$

$dx = 4\sec\theta\tan\theta\, d\theta$

$$\therefore\ I = \int \frac{128 \cdot 4\sec\theta\tan\theta}{64\sec^3\theta\sqrt{4^2\sec^2\theta - 4^2}}\, d\theta$$

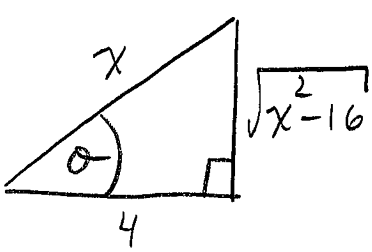
$$= \frac{\overset{2}{\cancel{(128)}}\cancel{(4)}}{\cancel{(64)}\cancel{(4)}}\int \frac{\sec\theta\ \cancel{\tan\theta}}{\sec^3\theta\ \cancel{\tan\theta}}\, d\theta$$

$$= 2\int \cos^2\theta\, d\theta$$

$$= \cancel{2}\int \frac{1+\cos(2\theta)}{\cancel{2}}\, d\theta$$

$$= \theta + \frac{1}{2}\sin(2\theta) + C$$

$$= \theta + \sin(\theta)\cos(\theta) + C$$



$$\boxed{\therefore\ I = \sec^{-1}\left(\frac{x}{3}\right) + \frac{\sqrt{x^2-16}}{x} \cdot \frac{4}{x} + C}$$

Question 5 [10 points]: Determine $\int \frac{1}{x^3+9x^2}\,dx = \int \frac{1}{x^2(x+9)}\,dx = I$

$$\frac{1}{x^2(x+9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+9}$$

$$= \frac{Ax(x+9) + B(x+9) + Cx^2}{x^2(x+9)}$$

$$= \frac{(A+C)x^2 + (9A+B)x + 9B}{x^2(x+9)}.$$

$\therefore$ $A + C = 0$

$9A + B = 0$

$9B = 1 \Rightarrow B = \frac{1}{9}$

$\therefore A = -\frac{B}{9} = \frac{-1}{81}$

$C = -A = \frac{1}{81}$

$$\therefore I = \int \frac{(-1/81)}{x} + \frac{(1/9)}{x^2} + \frac{(1/81)}{x+9}\,dx$$

$$= \boxed{-\frac{1}{81}\ln|x| - \frac{1}{9}\left(\frac{1}{x}\right) + \frac{1}{81}\ln|x+9| + C}$$