Question 1:

(a)[2] Determine $\int_0^1 \frac{d}{dx}[e^{\arctan(x)}]\,dx$. $= \left[e^{\arctan(x)}\right]_0^1 = e^{\frac{\pi}{4}} - e^0 = \boxed{e^{\frac{\pi}{4}} - 1}$

(b)[2] Determine $\frac{d}{dx}\left[\int_0^x e^{\arctan(t)}\,dt\right]$. $= \boxed{e^{\arctan(x)}}$ by F.T.C.

(c)[2] Determine $\frac{d}{dx}\left[\underbrace{\int_0^1 e^{\arctan(x)}\,dx}_{\text{a constant}}\right]$. $= \boxed{0}$

(d)[4] Determine $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$. let $u = x^{\frac{1}{2}}$, $du = \frac{1}{2}x^{-\frac{1}{2}}dx$

$\therefore I = 2\int \cos(u)\,du$

$= 2\sin(u) + C$

$= \boxed{2\sin(\sqrt{x}) + C}$

Question 2:

(a)[4] Determine $\int (t+1)\ln(t)\,dt$. $\left\{ \begin{array}{l} u = \ln(t) ; \ dv = (t+1)\,dt \\ du = \frac{1}{t}\,dt ; \ v = \frac{t^2}{2} + t \end{array} \right.$

$$\therefore I = \int u\,dv = uv - \int v\,du$$
$$= \ln(t) \cdot \left(\frac{t^2}{2} + t\right) - \int \left(\frac{t^2}{2} + t\right)\left(\frac{1}{t}\right) dt$$
$$= \ln(t)\left(\frac{t^2}{2} + t\right) - \int \frac{1}{2}t + 1 \; dt$$
$$= \boxed{\ln(t)\left(\frac{t^2}{2} + t\right) - \left(\frac{1}{4}t^2 + t\right) + C}$$

(b)[6] Determine $\int e^{-x}\sin(x)\,dx = I$

$u = e^{-x}$; $dv = \sin(x)\,dx$
$du = -e^{-x}\,dx$; $v = -\cos(x)\,dx$

$$\therefore I = \int u\,dv = -e^{-x}\cos(x) - \underbrace{\int \cos(x) e^{-x}\,dx}_{\substack{\text{let } u = e^{-x};\ dv = \cos(x)dx \\ du = -e^{-x}dx;\ v = \sin(x)}}$$

$$\therefore I = -e^{-x}\cos(x) - \left[e^{-x}\sin(x) + \underbrace{\int \sin(x) e^{-x}\,dx}_{I} \right]$$

$$\therefore I = -e^{-x}\cos(x) - e^{-x}\sin(x) - I$$

$$\therefore I = \boxed{\frac{-e^{-x}\left(\cos(x) + \sin(x)\right)}{2} + C}$$

Question 3:

(a)[7] Evaluate $\int_0^{\pi/2} \cos^3(x)\sin^3(x)\,dx$. $= \int_0^{\frac{\pi}{2}} \sin^3(x)\left(1-\sin^2(x)\right)\cos(x)\,dx.$

Let $u = \sin(x)$, $du = \cos(x)\,dx.$ $\left.\begin{matrix} x=0 \Rightarrow u=\sin(0)=0 \\ x=\frac{\pi}{2} \Rightarrow u=\sin(\frac{\pi}{2})=1 \end{matrix}\right\}$

$$\therefore I = \int_0^1 u^3(1-u^2)\,du$$

$$= \left[\frac{u^4}{4} - \frac{u^6}{6}\right]_0^1$$

$$= \frac{1}{4} - \frac{1}{6}$$

$$= \boxed{\frac{1}{12}}$$

(b)[3] Determine $\int \cot(\pi x)\,dx$. $= \int \frac{\cos(\pi x)}{\sin(\pi x)}\,dx = I$

let $u = \sin(\pi x)$

$du = \cos(\pi x)\,\pi\,dx$

$$\therefore I = \frac{1}{\pi}\int \frac{1}{u}\,du$$

$$= \frac{1}{\pi}\ln|u| + C = \boxed{\frac{1}{\pi}\ln\left|\sin(\pi x)\right| + C}$$

Question 4 [10 points]: Determine $\int \frac{54}{x^3\sqrt{x^2-9}}\,dx$.

let $x = 3\sec\theta \rightarrow \sec\theta = \frac{x}{3}$

$dx = 3\sec\theta\tan\theta\,d\theta$

$$\therefore I = \int \frac{54 \cdot 3\sec\theta\tan\theta\,d\theta}{27\sec^3\theta\sqrt{9\sec^2\theta - 9}}$$

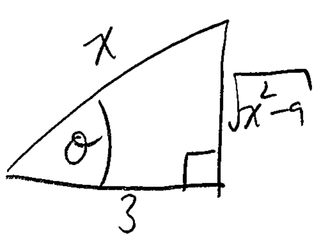
$$= \frac{(54)\cancel{(3)}}{(27)\cancel{(3)}} \int \frac{\sec\theta\,\cancel{\tan\theta}}{\sec^3\theta\,\cancel{\tan\theta}}\,d\theta$$

$$= 2\int \cos^2\theta\,d\theta$$

$$= \cancel{2}\int \frac{1+\cos(2\theta)}{\cancel{2}}\,d\theta$$

$$= \theta + \frac{1}{2}\sin(2\theta) + C$$

$$= \theta + \sin(\theta)\cos(\theta) + C.$$



$$\therefore I = \boxed{\sec^{-1}\left(\frac{x}{3}\right) + \frac{\sqrt{x^2-9}}{x}\cdot\frac{3}{x} + C}$$

Question 5 [10 points]: Determine $\int \frac{1}{x^3+4x^2}\,dx$. $= \int \frac{1}{x^2(x+4)}\,dx = I$

$$\frac{1}{x^2(x+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4}$$

$$= \frac{Ax(x+4) + B(x+4) + Cx^2}{x^2(x+4)}$$

$$= \frac{(A+C)x^2 + (4A+B)x + 4B}{x^2(x+4)}$$

$\therefore$ $A+C=0$

$4A+B=0$

$\left.4B=1\right\} \longrightarrow B = \frac{1}{4}$

$\therefore A = \frac{-B}{4} = \frac{-1}{16}$

$\therefore C = -A = \frac{1}{16}$.

$$\therefore I = \int -\frac{1}{16}\frac{1}{x} + \frac{1}{4}\frac{1}{x^2} + \frac{1}{16}\frac{1}{x+4}\,dx$$

$$= \boxed{-\frac{1}{16}\ln|x| - \frac{1}{4}\cdot\frac{1}{x} + \frac{1}{16}\ln|x+4| + C}$$