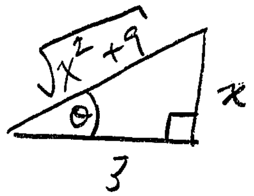
Question 1:

(a)[3] Write as a simplified expression which does not involve trigonometric functions:

$$\cos\left[\tan^{-1}\left(\frac{x}{3}\right)\right]$$

$\theta = \tan^{-1}\left(\frac{x}{3}\right)$:



$$\therefore \cos\left[\tan^{-1}\left(\frac{x}{3}\right)\right] = \cos\theta = \boxed{\frac{3}{\sqrt{x^2+9}}}$$

(b)[3] Determine $f'(0)$ where

$$f(x) = \sin\left(e^x - 1\right)\arcsin\left(e^x - 1\right)$$

$$f'(x) = \cos(e^x-1)\cdot e^x \cdot \arcsin(e^x-1) + \sin(e^x-1)\frac{e^x}{\sqrt{1-(e^x-1)^2}}$$

$$f'(0) = \underbrace{\cos(0)\cdot e^0 \cdot \arcsin(0)}_{\to 0} + \underbrace{\sin(0)\frac{e^0}{\sqrt{1-0^2}}}_{\to 0}$$

$$= \boxed{0}$$

(c)[4] Determine the exact value of

$$\sin\left(\arcsin\left(\sqrt{3}/2\right) - \arccos\left(\sqrt{3}/2\right)\right)$$

$$= \sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \sin\left(\frac{\pi}{6}\right)$$

$$= \boxed{\frac{1}{2}}$$

Question 2:

(a)[5] Find all values of x for which $\cosh(\ln x) = 1$.

$$\cosh(\ln x) = 1$$

$$\frac{e^{\ln(x)} + e^{-\ln(x)}}{2} = 1$$

$$e^{\ln(x)} + e^{\ln(x^{-1})} = 2$$

$$x + x^{-1} = 2$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$\boxed{x = 1}$$

(b)[5] Determine $f(x)$ if

$$f''(x) = 15\sqrt{x} + \sinh(x), \quad f'(0) = 1, f(0) = \pi$$

$$f''(x) = 15x^{1/2} + \sinh(x)$$

$$f'(x) = 15\frac{x^{3/2}}{(3/2)} + \cosh(x) + C_1$$

$$= 10x^{3/2} + \cosh(x) + C_1$$

$$f'(0) = 1 \Rightarrow 0 + 1 + C_1 = 1$$

$$\therefore C_1 = 0$$

$$\therefore f'(x) = 10x^{3/2} + \cosh(x)$$

$$f(x) = 4x^{5/2} + \sinh(x) + C_2$$

$$f(0) = \pi \Rightarrow 0 + 0 + C_2 = \pi$$

$$\therefore f(x) = 4x^{5/2} + \sinh(x) + \pi$$

Question 3:

(a)[5] Evaluate the limit if it exists:

$$\lim_{x\to 0^+} x^2 \ln(x) \quad \sim \text{"}0 \cdot -\infty\text{"}$$

$$= \lim_{x \to 0^+} \frac{\ln(x)}{\frac{1}{x^2}} \quad \sim \text{"}\frac{-\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$$

$$= \lim_{x \to 0^+} -\frac{1}{2} x^2$$

$$= \boxed{0}$$

(b)[5] Evaluate the limit if it exists:

$$\lim_{x\to \pi/2} \frac{e^{\cos x} - 1}{x - \pi/2} \quad \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x \to \frac{\pi}{2}} \frac{-\sin x \, e^{\cos x}}{1}$$

$$= -1 \cdot e^0$$

$$= \boxed{-1}$$

Question 4:

(a)[5] Evaluate the limit if it exists:

$$\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right) \sim \text{"}\infty - \infty\text{"}$$

$$= \lim_{x\to 1^+} \frac{x\ln x - x + 1}{(x-1)\ln(x)} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 1^+} \frac{\ln(x) + \cancel{x\frac{1}{x}} \cancel{-1}}{\ln(x) + (x-1)\left(\frac{1}{x}\right)} \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 1^+} \frac{\frac{1}{x}}{\frac{1}{x} + \frac{1}{x} + (x-1)\left(\frac{-1}{x^2}\right)}$$

$$= \boxed{\frac{1}{2}}$$

(b)[5] Evaluate the limit if it exists:

$$\lim_{x\to 1^+} x^{1/(x-1)} \sim \text{"}1^\infty\text{"}$$

$x^{\frac{1}{x-1}} = e^{\left(\frac{1}{x-1}\right)\ln x}$; consider $\lim_{x\to 1^+} \frac{\ln x}{x-1} \sim \text{"}\frac{0}{0}\text{"}$

$$\overset{H}{=} \lim_{x\to 1^+} \frac{\frac{1}{x}}{1}$$

$$= 1$$

$\therefore \lim_{x\to 1^+} x^{\frac{1}{x-1}} = e^1 = \boxed{e}$

Question 5: Use the definition of the definite integral in the form

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

to evaluate

$$\int_0^2 \left(2x^3 - 3x\right)\,dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplication.

$$f(x) = 2x^3 - 3x$$

$$[a,b] = [0,2]$$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$$

$$x_i = a + i\Delta x = 0 + i\left(\frac{2}{n}\right) = \frac{2i}{n}.$$

$$\therefore \int_0^2 (2x^3 - 3x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(\frac{2i}{n}\right)^3 - 3\left(\frac{2i}{n}\right)\right]\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{32\,i^3}{n^4} - \frac{12\,i}{n^2}\right)$$

$$= \lim_{n\to\infty} \left[\frac{32}{n^4}\left(\sum_{i=1}^{n} i^3\right) - \frac{12}{n^2}\left(\sum_{i=1}^{n} i\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{32}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{12}{n^2}\,\frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty} \left[8\,\cancel{\frac{n}{n}}\cdot\cancel{\frac{n}{n}}\cdot\frac{(n+1)}{n}\cdot\frac{(n+1)}{n} - 6\,\cancel{\frac{n}{n}}\,\frac{n+1}{n}\right]$$

$$= \boxed{2}$$