

Question 1:

(a)[3] Write as a simplified expression which does not involve trigonometric functions:

$$\cos \left[\tan^{-1} \left(\frac{x}{3} \right) \right]$$

(b)[3] Determine $f'(0)$ where

$$f(x) = \sin(e^x - 1) \arcsin(e^x - 1)$$

(c)[4] Determine the exact value of

$$\sin \left(\arcsin(\sqrt{3}/2) - \arccos(\sqrt{3}/2) \right)$$

Question 2:

(a)[5] Find all values of x for which $\cosh(\ln(x)) = 1$.

(b)[5] Determine $f(x)$ if

$$f''(x) = 15\sqrt{x} + \sinh(x), \quad f'(0) = 1, f(0) = \pi$$

Question 3:

(a)[5] Evaluate the limit if it exists:

$$\lim_{x \rightarrow 0^+} x^2 \ln x$$

(b)[5] Evaluate the limit if it exists:

$$\lim_{x \rightarrow \pi/2} \frac{e^{\cos x} - 1}{x - \pi/2}$$

Question 4:

(a)[5] Evaluate the limit if it exists:

$$\lim_{x \rightarrow 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$$

(b)[5] Evaluate the limit if it exists:

$$\lim_{x \rightarrow 1^+} x^{1/(x-1)}$$

Question 5: Use the definition of the definite integral in the form

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

to evaluate

$$\int_0^2 (2x^3 - 3x) dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplification.