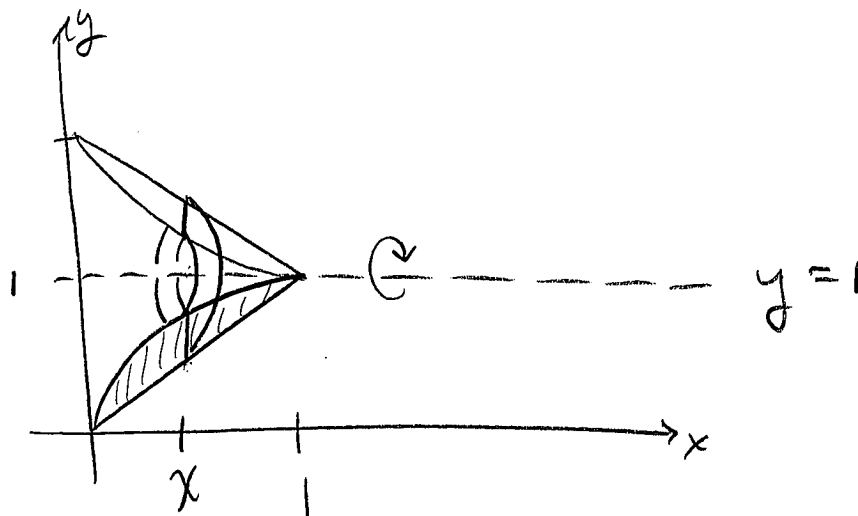


(1) [7] The region bounded by the curves $y = x$ and $y = \sqrt{x}$ is rotated about the line $y = 1$. Determine the volume of the resulting solid. (The disk method would work best here.)



$$A(x) = \pi (1-x)^2 - \pi (1-\sqrt{x})^2$$

$$\therefore V = \int_0^1 \pi (1-x)^2 - \pi (1-\sqrt{x})^2 dx$$

$$= \pi \int_0^1 x - 2x + x^2 + 2\sqrt{x} - x dx$$

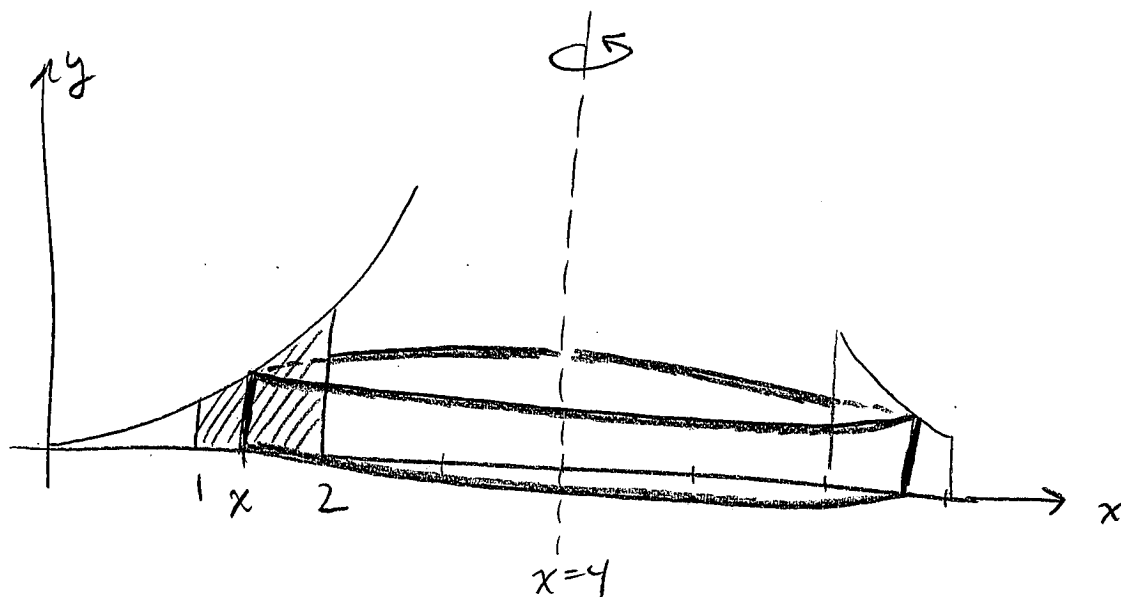
$$= \pi \left[\frac{x^3}{3} - 3 \frac{x^2}{2} + 2 \frac{x^{3/2}}{3/2} \right]_0^1$$

$$= \pi \left[\frac{1}{3} - \frac{3}{2} + \frac{4}{3} \right]$$

$$= \pi \left[\frac{2-9+8}{6} \right]$$

$$= \boxed{\frac{\pi}{6}}$$

(2) [8] The region bounded by the curves $y = x^2$, $y = 0$, $x = 1$ and $x = 2$ is rotated about the line $x = 4$. Determine the volume of the resulting solid. (Cylindrical shells would be best here.)



$$\begin{aligned}
 V &= \int_1^2 2\pi (4-x) x^2 dx \\
 &= 2\pi \int_1^2 (4x^2 - x^3) dx \\
 &= 2\pi \left[\frac{4}{3} x^3 - \frac{x^4}{4} \right]_1^2 \\
 &= 2\pi \left[\left(\frac{32}{3} - 4 \right) - \left(\frac{4}{3} - \frac{1}{4} \right) \right] \\
 &= 2\pi \frac{128 - 48 - 16 + 3}{6} \\
 &= \frac{67\pi}{6}
 \end{aligned}$$