

(1) [7] Evaluate the improper integral

$$I = \int_{-2}^{14} \frac{1}{\sqrt[4]{x+2}} dx$$

making proper use of any required limits.

$$I = \lim_{t \rightarrow -2^+} \int_t^{14} (x+2)^{-\frac{1}{4}} dx$$

$$= \lim_{t \rightarrow -2^+} \frac{4}{3} \left[(x+2)^{\frac{3}{4}} \right]_t^{14}$$

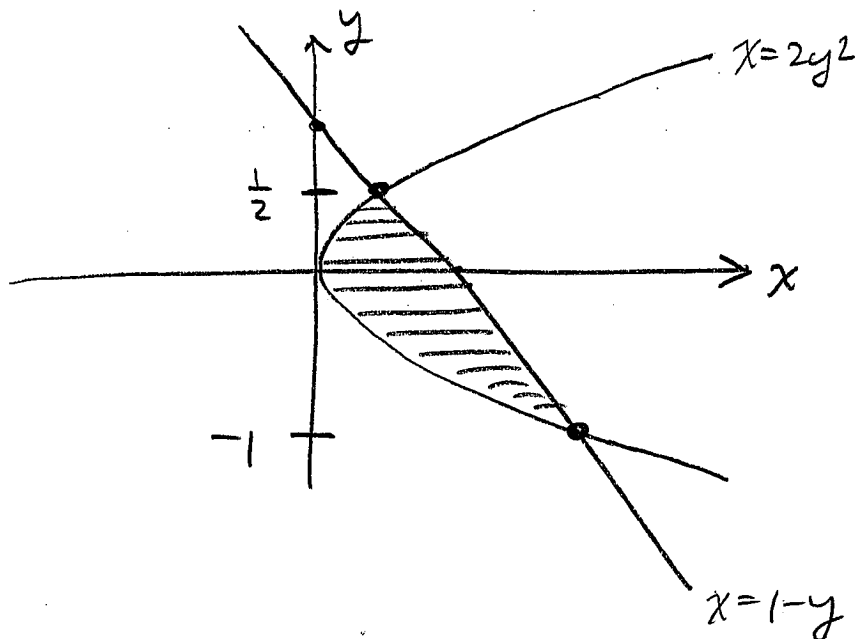
$$= \lim_{t \rightarrow -2^+} \frac{4}{3} \left[16^{\frac{3}{4}} - (t+2)^{\frac{3}{4}} \right]$$

$$= \left(\frac{4}{3} \right) \left[\left(16^{\frac{1}{4}} \right)^3 - 0 \right]$$

$$= \frac{4^4}{3}$$

$$= \boxed{\frac{256}{3}}$$

(2) [8] Determine the area enclosed by the curves $x = 2y^2$ and $x + y = 1$.



$$\left. \begin{array}{l} x = 2y^2 \\ x + y = 1 \end{array} \right\} \Rightarrow \begin{aligned} 2y^2 + y - 1 &= 0 \\ y &= \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)} \\ &= \frac{-1 \pm 3}{4} \\ &= \frac{1}{2}, -1 \end{aligned}$$

∴ area $A = \int_{y=-1}^{y=1/2} (1-y) - (2y^2) dy$

$$= \left[y - \frac{y^2}{2} - \frac{2}{3} y^3 \right]_{-1}^{1/2}$$

$$= \left(\frac{1}{2} - \frac{1}{8} - \frac{1}{12} \right) - \left(-1 - \frac{1}{2} + \frac{2}{3} \right)$$

$$= \frac{12 - 3 - 2 + 24 + 12 - 16}{24} = \boxed{\frac{27}{24}}$$