Math 122 - Quiz 4 Name: SOLUTIONS.

(1) [7] Determine

$$I = \int \tan^5(x)\sec^4(x)\,dx$$

$$= \int \tan^4(x)\sec^3(x)\ \sec(x)\tan(x)\,dx$$

$$= \int (\tan^2(x))^2\sec^3(x)\ \sec(x)\tan(x)\,dx$$

$$= \int (\sec^2(x)-1)^2\sec^3(x)\ \sec(x)\tan(x)\,dx$$

$$u = \sec(x),\quad du = \sec(x)\tan(x)\,dx$$

$$\therefore\ I = \int (u^2-1)^2 u^3\,du$$

$$= \int u^7 - 2u^5 + u^3\,du$$

$$= \frac{u^8}{8} - 2\frac{u^6}{6} + \frac{u^4}{4} + C$$

$$= \boxed{\frac{\sec^8(x)}{8} - \frac{1}{3}\sec^6(x) + \frac{\sec^4(x)}{4} + C}$$

OR:
$$I = \int \tan^5(x)\sec^2(x)\sec^2(x)\,dx$$

$$= \int \tan^5(x)(1+\tan^2(x))\sec^2(x)\,dx \quad \left\}\begin{array}{l} u = \tan(x) \\ du = \sec^2(x)\,dx \end{array}\right.$$

$$= \int u^5(1+u^2)\,du$$

$$= \frac{u^6}{6} + \frac{u^8}{8} + C$$

$$= \boxed{\frac{1}{6}\tan^6(x) + \frac{1}{8}\tan^8(x) + C}$$

(2) [8] Determine

$$I = \int \frac{1}{x^2\sqrt{25-x^2}}\,dx$$

Let $x = 5\sin\theta$

$dx = 5\cos\theta\,d\theta.$

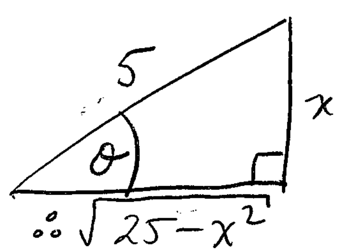
$$\therefore I = \int \frac{1}{25\sin^2\theta\sqrt{25-25\sin^2\theta}}\,5\cos\theta\,d\theta$$

$$= \frac{1}{25}\int \frac{\cancel{\cos\theta}}{\sin^2\theta\cdot\cancel{\cos\theta}}\,d\theta$$

$$= \frac{1}{25}\int \csc^2\theta\,d\theta$$

$$= -\frac{1}{25}\cot\theta + C.$$

$\sin\theta = \frac{x}{5}$



$$\boxed{\therefore I = \frac{-1}{25}\cdot\frac{\sqrt{25-x^2}}{x} + C}$$