Question 1 [10 points]: Evaluate

$$\int \frac{8x}{(x-2)(x+2)^2}\,dx$$

Question 2:

(a)[6] Use T_4, the trapezoid rule on 4 subintervals, to approximate $\int_{-1}^{1} 2e^{\sin(\pi x)}\,dx$.

(b)[4] For the integrand $f(x) = 2e^{\sin(\pi x)}$ in part (a),

$$f''(x) = 2\pi^2 e^{\sin(\pi x)}\left[\cos^2(\pi x) - \sin(\pi x)\right] .$$

On the interval $[-1, 1]$ this second derivative has an absolute maximum at $x = 0$ and an absolute minimum at $x = 1/2$. Use this information to determine an error bound on your approximation in part (a).

Recall: the error in using the trapezoid rule to approximate $\int_a^b f(x)\,dx$ using n subintervals is at most $\frac{K(b-a)^3}{12n^2}$ where $|f''(x)| \le K$ on $[a, b]$.

Question 3:

(a)[5] Evaluate the improper integral. Clearly show all steps including any required limits, and state, based on your answer, whether the integral converges or diverges:

$$\int_0^\infty \frac{e^x}{e^{2x}+1}\,dx$$

(b)[5] Determine (with justification) whether $\displaystyle\int_1^\infty \frac{\cos^4 x + 4}{x^4}\,dx$ converges or diverges.

Question 4:

(a)[5] Determine the area of the region bounded by the two parabolas $x = 2y^2$ and $x = 4 + y^2$:

(b)[5] The base (flat bottom) of a solid is the region in the xy-plane bounded by the curves $y = e^{-x}$, $x = 0$ and $x = 1$. Cross-sections perpendicular to the x-axis are semicircles. Determine the volume of the solid.

Question 5:

(a)[5] The region bounded by the curves $y = 2x$, $y = x^2/2$ and $y = 2$ is rotated about the y-axis. Determine the volume of the resulting solid.

(b)[5] Determine the length of the curve $y = 2\ln(\cos(x/2))$, $0 \leq x \leq \pi/2$.