Question 1: Recall Newton's Law of Cooling and Heating: if $T(t)$ is the temperature of an object at time t and T_S is the temperature of the surroundings, then

$$\frac{dT}{dt} = k(T - T_S) \ ,$$

and

$$T(t) = T_S + (T_0 - T_S)e^{kt} \ .$$

Here k is a negative constant and $T_0 = T(0)$, the initial temperature of the object.

(a)[5] A cup of coffee has an initial temperature of 95°C in a 20°C room. At the instant that the cup of coffee reaches 70°C it is cooling at a rate of 1°C per minute. Determine the value of the constant k using this information.

(b)[5] Using your result from part (a), determine the time at which the cup of coffee reaches the 70°C temperature.

Question 2:

(a)[3] Write as a simplified expression which does not involve trigonometric or inverse trigonometric functions:

$$\cot\left(\arcsin\left(x/3\right)\right)$$

(b)[3] Determine the exact value of $\cos^{-1}(-1/2)$.

(c)[4] Let $f(x) = \arccos\left(\arcsin\left(x^2\right)\right)$. Compute $f'(0)$.

Question 3:

(a)[3] Evaluate $\lim_{x\to\infty} e^{-2x}\sinh(2x)$.

(b)[3] Are there any values of $x > 0$ for which tangent lines to $y = \dfrac{\sinh^2(x)}{2} - \cosh(x)$ are horizontal? Explain.

(c)[4] Solve for x: $\sinh(x) = 1$.

Question 4:

(a)[5] Evaluate $\lim_{x\to 0} \dfrac{1-\cos(x)}{x^2+x}$

(b)[5] Evaluate $\lim_{x\to 1^+} \left(\dfrac{x}{x-1} - \dfrac{1}{\ln x}\right)$

Question 5:

(a)[5] Evaluate $\lim_{x\to\infty} x\tan(1/x)$

(b)[5] Evaluate $\lim_{x\to 0^+} x^{\sqrt{x}}$