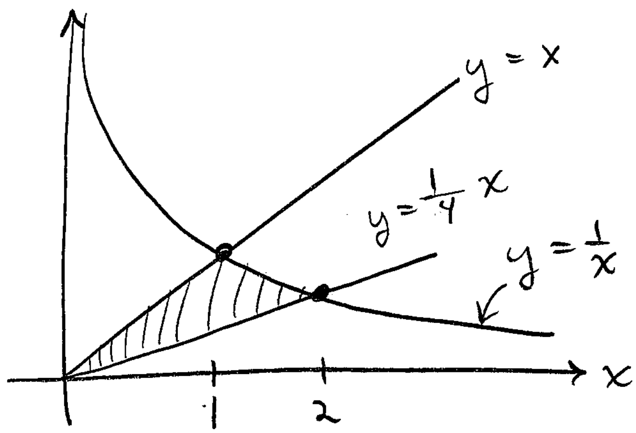
(1) [7] Determine the area of the region in the first quadrant that is bounded by the curves

$$y = \frac{1}{x}, \quad y = x, \quad y = \frac{1}{4}x$$



Solving $\left.\begin{array}{l} y = x \\ y = \frac{1}{x} \end{array}\right\} \Rightarrow x = \frac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = 1$

Solving $\left.\begin{array}{l} y = \frac{1}{4}x \\ y = \frac{1}{x} \end{array}\right\} \Rightarrow \frac{1}{4}x = \frac{1}{x} \Rightarrow x^2 = 4 \Rightarrow x = 2$

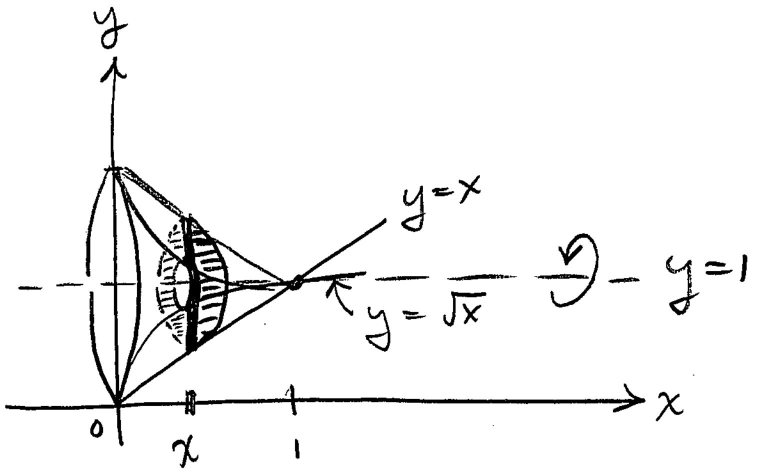
$\therefore$ area $A = \int_0^1 \left(x - \frac{1}{4}x\right)dx + \int_1^2 \left(\frac{1}{x} - \frac{1}{4}x\right)dx$

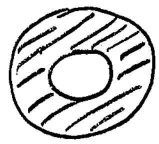
$$= \frac{3}{4}\left[\frac{x^2}{2}\right]_0^1 + \left[\ln|x| - \frac{1}{4}\frac{x^2}{2}\right]_1^2$$

$$= \left(\frac{3}{4}\right)\left(\frac{1}{2}\right) + \ln 2 - \frac{1}{2} + \frac{1}{8}$$

$$= \boxed{\ln 2}$$

(2) [8] The region in the first quadrant bounded by the curves $y = x$ and $y = \sqrt{x}$ is rotated about the line $y = 1$. Determine the volume of the resulting solid.



$A(x)$ = area of 

$$= \pi(1-x)^2 - \pi(1-\sqrt{x})^2$$

$$= \pi\left[\cancel{1} - 2x + x^2 - \cancel{1} + 2x^{1/2} - x\right]$$

$$= \pi\left[x^2 - 3x + 2x^{1/2}\right].$$

$$\therefore \; V = \int_0^1 A(x)\,dx$$

$$= \int_0^1 \pi\left[x^2 - 3x + 2x^{1/2}\right] dx$$

$$= \pi\left[\frac{x^3}{3} - 3\frac{x^2}{2} + \frac{2x^{3/2}}{3/2}\right]_0^1$$

$$= \pi\left[\frac{1}{3} - \frac{3}{2} + \frac{4}{3}\right]$$

$$= \pi\left(\frac{2-9+8}{6}\right)$$

$$= \boxed{\frac{\pi}{6}}$$