(1) [10] Determine

$$I = \int \frac{10}{(x-1)(x^2+9)}\,dx$$

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$$= \frac{A(x^2+9) + (Bx+C)(x-1)}{(x-1)(x^2+9)}$$

$$= \frac{Ax^2+9A+Bx^2+Cx-Bx-C}{(x-1)(x^2+9)}$$

$$= \frac{(A+B)x^2+(C-B)x+9A-C}{(x-1)(x^2+9)}$$

$\therefore$ $A+B=0 \Rightarrow A=-B$

$C-B=0 \Rightarrow C=B=-A$

$9A-C=10 \Rightarrow 9A+A=10 \Rightarrow A=1\,;\ \therefore C=-1\,;\ B=-1$

$$\therefore I = \int \frac{1}{x-1}\,dx + \int \frac{-x-1}{x^2+9}\,dx$$

$$= \int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{2x}{x^2+9}\,dx - \int \frac{1}{x^2+9}\,dx$$

$$= \boxed{\ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C}$$

(2) [5] Use T_2, the trapezoid rule on two subintervals, to estimate the value of

$$\int_e^{2e} \frac{12\ln x}{x}\,dx\,.$$

Simplify your answer.

$$\Delta x = \frac{b-a}{2} = \frac{2e-e}{2} = \frac{e}{2}$$

$$\therefore \int_e^{2e} \frac{12\ln x}{x}\,dx \approx T_2$$

$$= \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + f(x_2)\right]$$

$$= \left(\frac{\cancel{e}}{4}\right)\left[\frac{12\ln(e)}{\cancel{e}} + \frac{24\ln(\frac{3}{2}e)}{\frac{3}{2}\cancel{e}} + \frac{12\ln(2e)}{2\cancel{e}}\right]$$

$$= 3\ln(e) + 4\ln\left(\frac{3}{2}e\right) + \frac{3}{2}\ln(2e)$$

$$= 3\cancel{\ln(e)}^{1} + 4\left[\ln(3) - \ln(2) + \cancel{\ln(e)}^{1}\right] + \frac{3}{2}\left(\ln(2) + \cancel{\ln(e)}^{1}\right)$$

$$= 3 + 4\ln(3) - 4\ln(2) + 4 + \frac{3}{2}\ln(2) + \frac{3}{2}$$

$$= \frac{17}{2} + 4\ln(3) - \frac{5}{2}\ln(2)$$