(1) [5] Evaluate

$$I = \int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx$$

Let $u = \ln x$ $\quad x = e \Rightarrow u = \ln(e) = 1$

$du = \frac{1}{x}\,dx$ $\quad x = e^4 \Rightarrow u = \ln(e^4) = 4$

$$\therefore I = \int_1^4 u^{-\frac{1}{2}}\,du$$

$$= 2\left[u^{\frac{1}{2}}\right]_1^4$$

$$= 2(2-1) = \boxed{2}$$

(2) [5] Determine

$$I = \int \ln(2x+1)\,dx\,.$$

Let $u = \ln(2x+1)$, $dv = dx$

$du = \frac{2}{2x+1}\,dx$, $v = x$.

$$\therefore I = \int u\,dv = uv - \int v\,du$$

$$= x\ln(2x+1) - \int \frac{2x}{2x+1}\,dx$$

$$= x\ln(2x+1) - \int \frac{2x+1-1}{2x+1}\,dx$$

$$= x\ln(2x+1) - \int 1 - \frac{1}{2x+1}\,dx$$

$$= \boxed{x\ln(2x+1) - x + \frac{\ln(2x+1)}{2} + C}$$

(3) [5] Determine

$$\int_0^1 \frac{y}{e^{2y}}\,dy. = \int_0^1 y e^{-2y}\,dy$$

let $u = y \quad dv = e^{-2y}dy$

$du = dy \quad v = \dfrac{e^{-2y}}{-2}.$

$$\therefore \int_0^1 \frac{y}{e^{2y}}\,dy = \int_0^1 u\,dv$$

$$= uv\Big|_0^1 - \int_0^1 v\,du$$

$$= \left[y\cdot\frac{e^{-2y}}{-2}\right]_0^1 - \int_0^1 \frac{e^{-2y}}{-2}\,dy$$

$$= \frac{e^{-2}}{-2} - 0 - \frac{1}{4}\left[e^{-2y}\right]_0^1$$

$$= \frac{e^{-2}}{-2} - \frac{1}{4}\left[e^{-2} - 1\right]$$

$$= \boxed{\frac{1}{4} - \frac{3}{4}e^{-2}}$$