

(1) [5] Let

$$h(x) = \int_2^{1/x} \arctan(t) dt.$$

Determine  $h'(x)$ .

$$\begin{aligned} h'(x) &= \arctan\left(\frac{1}{x}\right) \cdot \frac{d}{dx}\left[\frac{1}{x}\right] \text{ by FTC 1} \\ &= \boxed{\arctan\left(\frac{1}{x}\right) \left(-\frac{1}{x^2}\right)}. \end{aligned}$$

(2) [5] Determine

$$I = \int e^x \sqrt{1+e^x} dx.$$

$$\begin{aligned} \text{let } u &= 1+e^x \\ du &= e^x dx \end{aligned}$$

$$\therefore I = \int u^{\frac{1}{2}} du$$

$$= \frac{2}{3} u^{\frac{3}{2}} + C$$

$$= \boxed{\frac{2}{3} (1+e^x)^{\frac{3}{2}} + C.}$$

(3) [5] Determine

$$I = \int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$$

$$\text{Let } u = x^{\frac{1}{2}}$$

$$du = \frac{1}{2} x^{-\frac{1}{2}} dx$$

$$x=1 \Rightarrow u = 1^{\frac{1}{2}} = 1$$

$$x=4 \Rightarrow u = 4^{\frac{1}{2}} = 2$$

$$\therefore I = 2 \int_1^2 e^u du$$

$$= 2 [e^u]_1^2$$

$$= \boxed{2(e^2 - e)}$$