 Name: SOLUTIONS

(1) [8] A bacteria culture initially contains 100 cells and grows at a rate that is proportional to its size. After an hour the population has increased to 400.

(a) Find an expression for $P(t)$, the number of bacteria after t hours. (Determine the value of any constants in your expression.)

model is $\frac{dP}{dt} = kP$, $P(0) = 100$, $P(1) = 400$.

$\therefore \quad P(t) = 100e^{kt}$

$P(1) = 400 \Rightarrow 400 = 100e^{k}$

$e^{k} = 4$

$k = \ln(4)$

$$\boxed{\therefore \; P(t) = 100e^{[\ln(4)]\cdot t}}$$

(b) When will the population reach 1000?

Solve $P(t) = 1000$

$100e^{[\ln(4)]t} = 1000$

$e^{[\ln(4)]t} = 10$

$\ln(4) \cdot t = \ln(10)$

$$\boxed{t = \frac{\ln(10)}{\ln(4)} \text{ hrs.}}$$

(2) [7] Find $g'(2)$ where

$$g(x) = x\sin^{-1}(x/4) + \sqrt{16 - x^2}$$

Simplify your answer, and in particular, determine the numerical value of any inverse trigonometric functions.

$$g'(x) = \sin^{-1}\left(\frac{x}{4}\right) + \frac{x}{\sqrt{1-\left(\frac{x}{4}\right)^2}} \cdot \frac{1}{4} + \frac{-\cancel{2}x}{\cancel{2}\sqrt{16-x^2}}$$

$$= \sin^{-1}\left(\frac{x}{4}\right) + \cancel{\frac{x}{\sqrt{16-x^2}}} - \cancel{\frac{x}{\sqrt{16-x^2}}}$$

$$\therefore\ g'(2) = \sin^{-1}\left(\frac{2}{4}\right) = \boxed{\frac{\pi}{6}}$$