Question 1:

(a)[5] Use differentials to estimate the amount of paint needed to apply a coat of paint $1/2000$ of a metre thick to a sphere of diameter 20 m.

Recall: the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

(b)[5] Use a linear approximation to estimate the value of $\sqrt{e^{0.1}}$.

Question 2:

(a)[5] Determine the inverse of the one-to-one function $y = \dfrac{1+e^x}{1-e^x}$.

(b)[5] Let $y = (\sin x)^{\ln x}$. Use logarithmic differentiation (or some other method) to find y' .

Question 3:

(a)[5] State the domain of $f(x) = \dfrac{x}{1 - \ln(x-1)}$ and determine $f'(2)$.

(b)[5] Determine all critical numbers of $g(x) = \dfrac{e^x}{1+x^2}$.

Question 4: For this question use the function $f(x) = x^3 + \frac{3}{x}$.

(a)[5] Determine the intervals of increase and decrease of f.

(b)[5] Determine the intervals of concavity of f.

Question 5 [10 points]: Sketch the graph of a function that has all of the following properties:

1. $f(-6) = 5$, $f(0) = 8$, $f(5) = 4$
2. $\lim_{x \to 8} f(x) = -\infty$
3. $f'(x) = 0$ at $x = -6$, $x = 0$ and $x = 5$
4. $f'(x) < 0$ on $(-\infty, -6) \cup (0, 5) \cup (5, 8)$
5. $f'(x) > 0$ on $(-6, 0) \cup (8, \infty)$
6. $f''(x) > 0$ on $(-\infty, -3) \cup (3, 5)$
7. $f''(x) < 0$ on $(-3, 3) \cup (5, 8) \cup (8, \infty)$

Indicate the scale on your graph and label all inflection points.