Question 1 [10 points]: Use the <u>definition of the derivative</u> to determine $f'(x)$ if $f(x) = \frac{5}{x+7}$. (No credit will be given if $f'(x)$ is found using derivative rules, though you may check your answer using the rules.)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{5}{x+h+7} - \frac{5}{x+7}\right]$$

$$= \lim_{h\to 0} \frac{5}{h}\left[\frac{\cancel{x}+\cancel{7} - \cancel{x} - h - \cancel{7}}{(x+h+7)(x+7)}\right]$$

$$= \lim_{h\to 0} \frac{5}{\cancel{h}}\left[\frac{-\cancel{h}}{(x+h+7)(x+7)}\right]$$

$$= \frac{-5}{(x+7)^2}$$

Question 2: Differentiate the following functions. It is not necessary to simplify final answers, but marks will be deducted for improper use of notation.

(a)[3 points] $f(x) = 3\sqrt{x} - \frac{2}{3x} + \pi^2$ $= 3x^{\frac{1}{2}} - \frac{2}{3}x^{-1} + \pi^2$

$$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + \frac{2}{3}x^{-2}$$

(b)[3 points] $y = (t^3 - 3\cos t)(t-1)^4$

$$y' = (3t^2 + 3\sin t)(t-1)^4 + (t^3 - 3\cos t)4(t-1)^3$$

(c)[4 points] $g(r) = \frac{r^2 - \sqrt[3]{r}}{\tan r}$ $= \frac{r^2 - r^{\frac{1}{3}}}{\tan r}$

$$g'(r) = \frac{(\tan r)(2r - \frac{1}{3}r^{-\frac{2}{3}}) - (r^2 - r^{\frac{1}{3}})\sec^2 r}{\tan^2 r}$$

Question 3: Differentiate the following functions. It is not necessary to simplify final answers, but marks will be deducted for improper use of notation.

(a)[3 points] $f(x) = \sin(x \cos x)$

$$f'(x) = \cos(x \cos x) \cdot \frac{d}{dx}\left[x \cos x\right]$$

$$= \cos(x\cos x)\left[\cos x - x \sin x\right]$$

(b)[3 points] $y = \sqrt{\pi^2 + \sec(2t)} = \left[\pi^2 + \sec(2t)\right]^{\frac{1}{2}}$

$$y' = \frac{1}{2}\left[\pi^2 + \sec(2t)\right]^{-\frac{1}{2}}\left[\sec(2t)\tan(2t)\right](2).$$

(c)[4 points] $g(\theta) = \tan(\theta - \sin(\sqrt{\theta})) = \tan\left(\theta - \sin\left(\theta^{\frac{1}{2}}\right)\right)$

$$g'(\theta) = \sec^2\left(\theta - \sin\left(\theta^{\frac{1}{2}}\right)\right) \cdot \frac{d}{d\theta}\left[\theta - \sin\left(\theta^{\frac{1}{2}}\right)\right]$$

$$= \sec^2\left(\theta - \sin\left(\theta^{\frac{1}{2}}\right)\right) \cdot \left[1 - \cos\left(\theta^{\frac{1}{2}}\right) \cdot \frac{1}{2}\theta^{-\frac{1}{2}}\right].$$

Question 4:

(a)[5 points] Determine the equation of the tangent line to the curve

$$y^3(x^2+y^2) = 1 + x^4 y$$

at the point $(1,1)$.

$$\frac{d}{dx}\left[y^3(x^2+y^2)\right] = \frac{d}{dx}\left[1+x^4y\right]$$

$$(3y^2y')(x^2+y^2) + y^3(2x+2yy') = 4x^3y + x^4y'$$

at $(1,1)$: $(3\cdot 1^2\cdot y')(1^2+1^2) + 1^3(2\cdot 1 + 2\cdot 1\cdot y') = 4\cdot 1^3\cdot 1 + 1^4\cdot y'$

$$6y' + 2 + 2y' = 4 + y'$$

$$7y' = 2$$

$$y' = \frac{2}{7}$$

$\therefore$ equation of tangent line is

$$\boxed{y - 1 = \frac{2}{7}(x-1)}$$

(b)[5 points] The position of a particle along a straight line at time t is given by

$$s(t) = \frac{t^3}{3} + qt(t+1)$$

where q is a constant. At time $t = 1$ the velocity and acceleration are the same. Determine the value of the constant q.

$$s(t) = \frac{1}{3}t^3 + qt^2 + qt$$

$$s'(t) = t^2 + 2qt + q$$

$$s''(t) = 2t + 2q$$

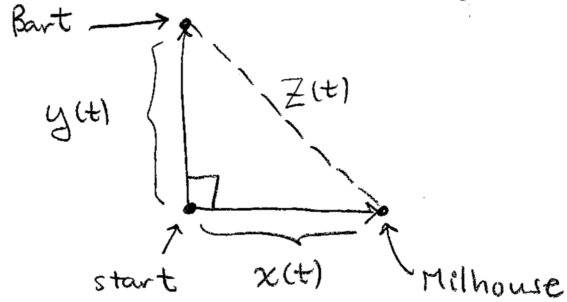
$$s'(1) = s''(1) \implies 1^2 + \cancel{2q\cdot 1} + q = 2\cdot 1 + \cancel{2q}$$

$$q = 2 - 1$$

$$\boxed{q = 1}$$

Question 5:

(a)[5 points] Bart and Milhouse begin walking from the same point at the same time. Bart walks north at 2 m/s, while Milhouse walks east at 1 m/s. At what rate is the distance between Bart and Milhouse increasing 5 seconds after they begin walking?



$$\frac{dy}{dt} = 2 \ \frac{m}{s}$$

$$\frac{dx}{dt} = 1 \ \frac{m}{s}.$$

Find $\frac{dz}{dt}$ at $t=5$.

$$z = (x^2+y^2)^{\frac{1}{2}} \implies \frac{dz}{dt} = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$

At $t=5$, $x=(1)(5)=5$, $y=(2)(5)=10$,

$$\therefore \left.\frac{dz}{dt}\right|_{t=5} = \frac{1}{2}(5^2+10^2)^{-\frac{1}{2}}\big((2)(5)(1)+(2)(10)(2)\big)$$

$$= \frac{50}{(2)\,5\sqrt{5}} = \sqrt{5}\ \frac{m}{s}.$$

$\therefore$ Distance between Bart and Milhouse is increasing at $\sqrt{5}\ \frac{m}{s}$.

(b)[5 points] At what values of x in the interval $[0, 2\pi]$ are tangent lines to the curve $y = \sin x + \cos x$ horizontal?

Solve $\frac{dy}{dx} = 0$: $\frac{dy}{dx} = \cos x - \sin x$;

$$\cos x - \sin x = 0$$

$$\sin x = \cos x$$

$$\therefore x = \frac{\pi}{4}, \frac{5\pi}{4}$$