

(1) [8] Determine the critical numbers of the function $f(x) = x^{4/5}(x-4)^2$.

$$\begin{aligned}
 f'(x) &= \frac{4}{5} x^{-\frac{1}{5}} (x-4)^2 + x^{\frac{4}{5}} \cdot 2(x-4) \\
 &= \frac{4(x-4)^2}{5x^{\frac{1}{5}}} + \frac{2x^{\frac{4}{5}}(x-4)}{1} \\
 &= \frac{4(x-4)^2 + 10x(x-4)}{5x^{\frac{1}{5}}} \\
 &= \frac{2(x-4) [2(x-4) + 5x]}{5x^{\frac{1}{5}}} \\
 &= \frac{2(x-4)(7x-8)}{5x^{\frac{1}{5}}}
 \end{aligned}$$

$$\begin{aligned}
 \underline{f'(x) = 0?} : \quad & x-4 = 0, \quad 7x-8 = 0 \\
 & x = 4, \quad x = \frac{8}{7}
 \end{aligned}$$

$$\underline{f'(x) \text{ not exist?}} : \quad x = 0$$

\therefore critical numbers are $x = 0, \frac{8}{7}, 4$.

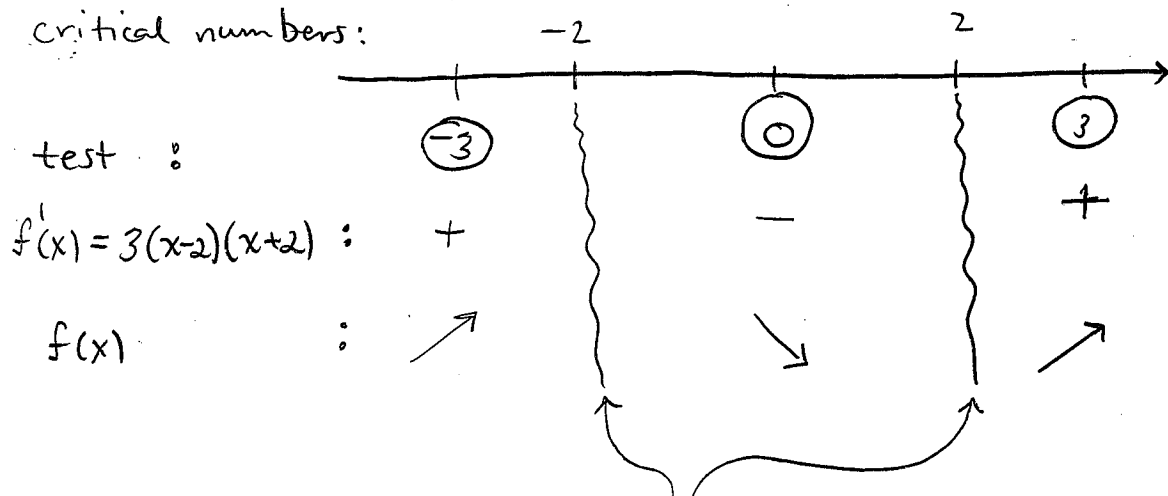
(2) [7] Determine the intervals of increase and decrease of $f(x) = x^3 - 12x + 1$. State a clear conclusion.

$$\begin{aligned} f'(x) &= 3x^2 - 12 \\ &= 3(x^2 - 4) \\ &= 3(x-2)(x+2) \end{aligned}$$

$$\begin{aligned} \underline{f'(x)=0?} \quad x-2=0, \quad x+2=0 \\ x=2, \quad x=-2. \end{aligned}$$

$$\underline{f'(x) \text{ not exist?}} \quad \text{no such } x.$$

critical numbers:



note: value of $f(x)$ at critical numbers not needed.

$\therefore f$ is increasing on $(-\infty, -2) \cup (2, \infty)$

f is decreasing on $(-2, 2)$.