Question 1:

(a)[5] Use matrix reduction to solve the following system of equations:

$$x - y + 2z = 6$$
$$2x + 2y - 4z = 0$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 2 & 2 & -4 & 0 \end{array}\right]$$

$R_2 = (-2)r_1 + r_2$:

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 0 & 4 & -8 & -12 \end{array}\right]$$

$R_2 = \frac{1}{4}r_2$:

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ 0 & 1 & -2 & -3 \end{array}\right]$$

$\therefore\ y - 2z = -3 \Rightarrow y = 2z - 3$

$x - y + 2z = 6 \Rightarrow x = y - 2z + 6$
$= 2z - 3 - 2z + 6$
$= 3$

$\therefore$ solution is

$\{(3, 2z-3, z) \mid z \text{ is any real number}\}$.

(b)[2] Is the system of equations in (a) consistent or inconsistent?

The system in (a) has at least one solution so it is consistent.

(c)[3] The row echelon form of a system of equations is

$$\left[\begin{array}{cc|c} 1 & -4 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

Does this system hava a solution? If so, state it using variables x and y; if not explain why.

Row 2 $\Rightarrow y = 1$

Row 1 $\Rightarrow x - 4y = 0$
$\Rightarrow x = 4y = 4$.

$\therefore$ Solution is $(4, 1)$.

Question 2: For this problem use the following matrices:

$$\mathbf{A} = \begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & -2 & 5 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} -2 & 1 \\ 3 & 5 \\ 5 & -2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} -3 & 4 \end{bmatrix}$$

(a)[4] Compute $3\mathbf{BC} + 2\mathbf{A}$

$$3BC + 2A = 3\begin{bmatrix} 2 & -1 & 1 \\ 0 & -2 & 5 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3 & 5 \\ 5 & -2 \end{bmatrix} + 2\begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} -2 & -5 \\ 19 & -20 \end{bmatrix} + 2\begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -9 \\ 63 & -58 \end{bmatrix}$$

(b)[2] What is the size (or dimension) of **EAD**?

$$\underbrace{E_{1\times 2}\; A_{2\times 2}\; D_{2\times 1}}_{1\times 1}$$

EAD has size 1×1.

(c)[4] Compute $\mathbf{A}(\mathbf{A}^{-1} + \mathbf{DE}) - \mathbf{I}_2$. Hint: you can do this calculation without determining $\mathbf{A}^{-1}$.

$$A\left(A^{-1} + DE\right) - I_2 = \cancel{I_2} + ADE - \cancel{I_2}$$

$$= \begin{bmatrix} -1 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \end{bmatrix}\begin{bmatrix} -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 8 \\ -12 & 16 \end{bmatrix}.$$

Question 3:

(a)[7] Determine $\mathbf{A}^{-1}$ where $\mathbf{A} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & -1 & 1 & 0 & 0 \\ 2 & -2 & -1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$r_1 \leftrightarrow r_3$:

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 1 & 0 & 0 & 1 \\ 2 & -2 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \end{array}\right]$$

$R_1 = (-1)$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 0 & -1 \\ 2 & -2 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \end{array}\right]$$

$R_2 = (-2)r_1 + r_2$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 & 0 & 0 \end{array}\right]$$

$r_2 \leftrightarrow r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 \end{array}\right]$$

$R_1 = (+1)r_2 + r_1$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & -1 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 \end{array}\right]$$

$R_1 = 2r_3 + r_1$:
$R_2 = 1 \cdot r_3 + r_2$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 & 2 \end{array}\right]$$

$\therefore A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

(b)[3] Use your result in part (a) to solve the following system of equations:

$$\begin{aligned} y - z &= 1 \\ 2x - 2y - z &= 0 \\ -x + y + z &= -2 \end{aligned}$$

System is $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$,

so $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -4 \end{bmatrix}$

$\therefore x = -5, \ y = -3, \ z = -4$

Question 4 [10 points]: A cookware factory has two machines for producing pots. Machine 1 can produce 60 large pots and 70 small pots each hour, while machine 2 can produce 40 large and 20 small pots each hour. Machine 1 costs \$70 per hour to operate and machine 2 only costs \$30 per hour. During each 10 hour day the factory must turn out at least 240 large pots and 140 small pots. How many hours should each machine be run each day in order to meet demand at the lowest cost?

Graph paper is provided on the next page. Carefully set up the problem, neatly sketch any required graphs and state a clear conclusion.

Let x = # hours machine 1 is run each day
y = # hours machine 2 is run each day.

We wish to minimize $Z = 70x + 30y$

subject to:

$$x \leq 10$$
$$y \leq 10$$
$$60x + 40y \geq 240$$
$$70x + 20y \geq 140$$
$$x \geq 0$$
$$y \geq 0$$

Corner points: by inspection: $(0,10)$, $(0,7)$, $(4,0)$, $(10,0)$
$(10,10)$.

Solving:

① $70x + 20y = 140$

② $60x + 40y = 240$

① $\Rightarrow$ $y = \dfrac{140 - 70x}{20} = 7 - \frac{7}{2}x$

② $\Rightarrow$ $60x + 40\left(7 - \frac{7}{2}x\right) = 240$

$60x + 280 - 140x = 240$

$-80x = -40$

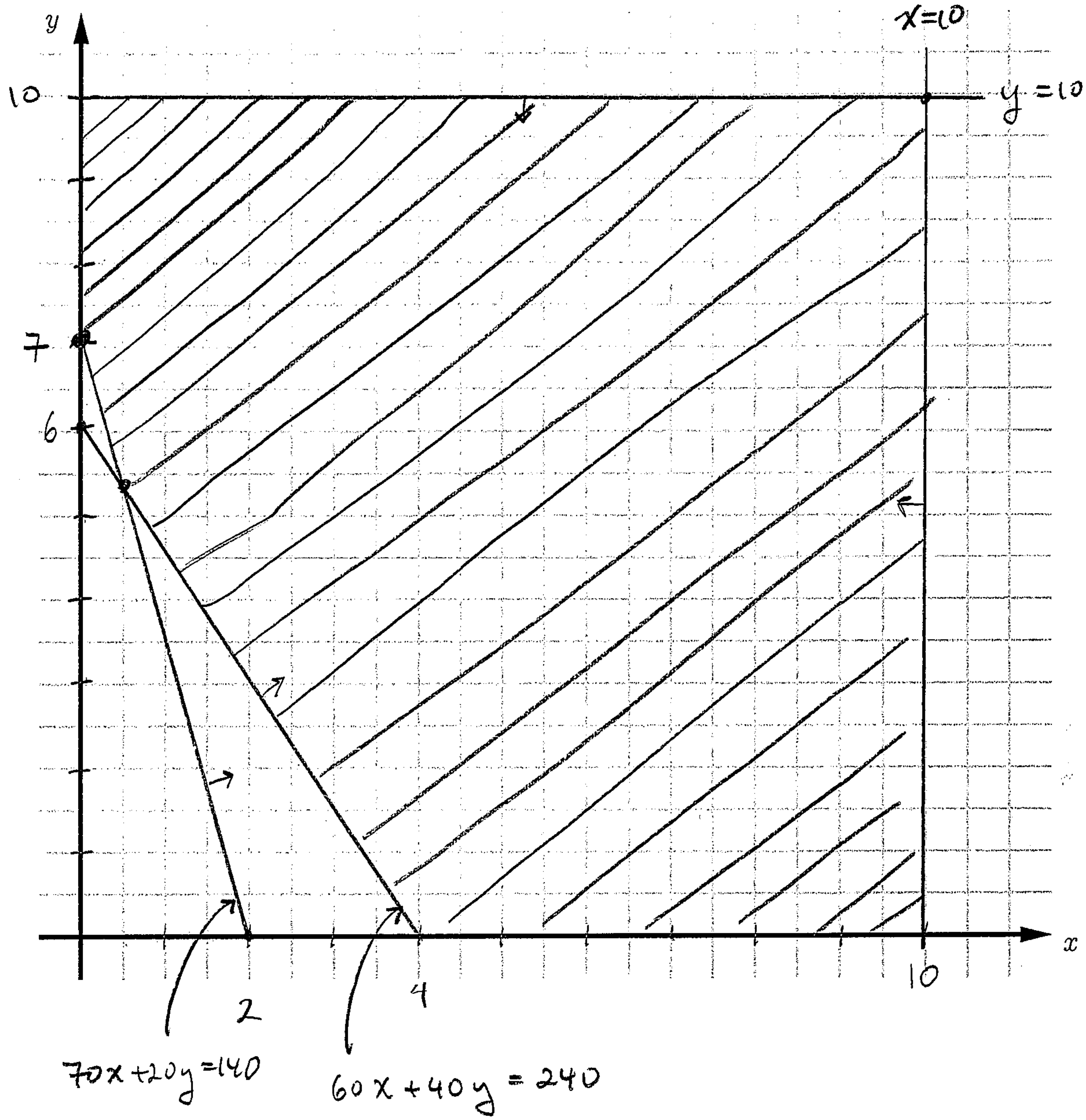
$x = \frac{1}{2}$

$\therefore$ $y = 7 - \left(\frac{7}{2}\right)\left(\frac{1}{2}\right) = \frac{21}{4}$

$\therefore \left(\frac{1}{2}, \frac{21}{4}\right)$

Cont'd. →

Question 4 (continued)



Corner points	$z = 70x + 30y$
(0,10)	300
(0,7)	210
(4,0)	280
(10,0)	700
(10,10)	1000
$(\frac{1}{2}, \frac{21}{4})$	192.50 ← min

∴ To minimize cost machine 1 should run for $\frac{1}{2}$ hour and machine 2 should run for $\frac{21}{4}$ hours.