

Question 1:

- (a)[2 points] Determine the slope and y-intercept of the line
- $3x - 2y = 5$
- .

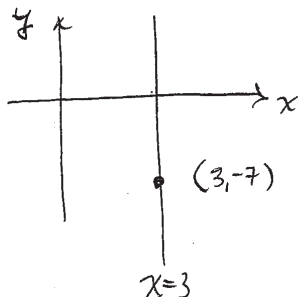
$$3x - 2y = 5$$

$$2y = 3x - 5$$

$$y = \frac{3}{2}x - \frac{5}{2}$$

$$\therefore m = \frac{3}{2}, \text{ y-intercept } (0, -\frac{5}{2})$$

- (b)[2 points] Determine an equation of the line through
- $(3, -7)$
- with slope undefined.



$$x=3$$

- (c)[3 points] Determine an equation of the line through the points
- $(7, 1)$
- and
- $(3, -7)$
- .

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-7)}{7 - 3} = \frac{8}{4} = 2$$

$$\therefore y - 1 = 2(x - 7)$$

$$\text{or } y = 2x - 13$$

- (d)[3 points] Determine an equation of the line that is parallel to the line
- $x - 2y = -3$
- and which passes through the point
- $(-3, 8)$
- .

$$x - 2y = -3$$

$$2y = x + 3$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

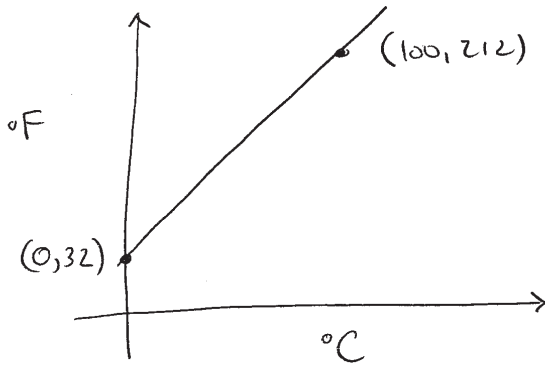
$$\therefore m = \frac{1}{2}$$

$$\therefore y - 8 = \frac{1}{2}(x + 3)$$

$$\text{or } y = \frac{1}{2}x + \frac{19}{2}$$

Question 2:

- (a)[5 points] The relationship between degrees Celsius ($^{\circ}C$) and degrees Fahrenheit ($^{\circ}F$) is linear. Find a linear relation (that is, the equation of the line) relating $^{\circ}C$ and $^{\circ}F$ if $0^{\circ}C$ corresponds to $32^{\circ}F$ and $100^{\circ}C$ corresponds to $212^{\circ}F$, and then use the relation you found to convert $68^{\circ}F$ to $^{\circ}C$.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

$$\therefore F - 32 = \frac{9}{5}(C - 0)$$

$$F = \frac{9}{5}C + 32$$

$$\text{If } F = 68,$$

$$68 = \frac{9}{5}C + 32$$

$$\frac{9}{5}C = 36$$

$$C = \left(\frac{5}{9}\right)(36)$$

$$C = 20$$

- (b)[5 points] Determine whether the following lines are parallel, coincident or intersecting:

$$L: 4x - 2y = -7$$

$$M: -2x + y = -1$$

$$\begin{aligned} L: 4x - 2y &= -7 \\ 2y &= 4x + 7 \\ y &= 2x + \frac{7}{2} \end{aligned}$$

$$\begin{aligned} M: -2x + y &= -1 \\ y &= 2x - 1 \end{aligned}$$

Since slopes are equal but y-intercepts differ, lines are parallel.

Question 3:

(a)[5 points] Determine the point of intersection of the lines

$$L: 4x + 3y = 2$$

$$M: 2x - y = 1$$

Using M: $y = 2x - 1$

Sub. \nearrow into L: $4x + 3(2x - 1) = 2$

$$4x + 6x - 3 = 2$$

$$10x = 5$$

$$x = \frac{1}{2}$$

$$\therefore y = 2\left(\frac{1}{2}\right) - 1 = 0$$

$$\therefore \boxed{\left(\frac{1}{2}, 0\right)}$$

(b)[5 points] A person has 20 coins which total \$1.65. If the coins consist of nickels (5¢ pieces) and dimes (10¢ pieces), how many of each type of coin does the person have? Clearly define your variables and state a clear conclusion.

Let $x =$ no. of nickels
 $y =$ no. of dimes.

$$\textcircled{1} \quad x + y = 20$$

$$\textcircled{2} \quad 5x + 10y = 165$$

$$\textcircled{1} \Rightarrow y = 20 - x$$

$$\textcircled{2} \Rightarrow 5x + 10(20 - x) = 165$$

$$5x + 200 - 10x = 165$$

$$5x = 35$$

$$x = 7$$

$$\therefore y = 20 - 7 = 13$$

\therefore The person has 7 nickels and 13 dimes.

Question 4:

- (a)[5 points] A certain product has supply equation $S = 40p + 300$ and a market price of \$30. Each \$4 increase in price reduces demand by 100 units. At what price does demand drop to 1175 units?

$$\text{At } p = 30, D = S = 40(30) + 300 = 1500.$$

$\therefore (30, 1500)$ is on D-line.

If $p = 30 + 4$, $D = 1500 - 100$; so $(34, 1400)$ is also on D-line.

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1500 - 1400}{30 - 34} = -25$$

$$\therefore D - 1500 = -25(p - 30)$$

$$\begin{aligned} \text{When } D = 1175 : \quad 1175 - 1500 &= -25(p - 30) \\ -325 &= -25p + 750 \\ p &= \frac{1075}{25} = 43 \end{aligned}$$

\therefore Demand drops to 1175 at $p = \$43$

- (b)[5 points] A ferry service has different fare options. One option is to pay a one-time fee of \$52 to join their frequent traveller club and then pay \$15 for every trip on the ferry. A second option is to simply pay a \$19 fare for each trip without joining the frequent traveller club. How many trips are required for both options to be equivalent in terms of cost?

Let x = the required number of trips.

We require

$$(\text{cost if member of club}) = (\text{regular cost})$$

$$52 + 15x = 19x$$

$$52 = 4x$$

$$x = 13$$

\therefore 13 trips are required.

Question 5 [10]: Solve the following system of equations **using matrix reduction** (no credit will be given for using any other method). Use proper notation to clearly state the row operations used at each step and clearly state the final solution.

$$3x - 3y - z = 1$$

$$-x + 2y + z = 5$$

$$3x - 4y - z = 1$$

$$\left[\begin{array}{ccc|c} 3 & -3 & -1 & 1 \\ -1 & 2 & 1 & 5 \\ 3 & -4 & -1 & 1 \end{array} \right]$$

$$\underline{r_1 \leftrightarrow r_2}: \left[\begin{array}{ccc|c} -1 & 2 & 1 & 5 \\ 3 & -3 & -1 & 1 \\ 3 & -4 & -1 & 1 \end{array} \right]$$

$$\underline{r_1 = (-1)r_1}: \left[\begin{array}{ccc|c} \textcircled{1} & -2 & -1 & -5 \\ 3 & -3 & -1 & 1 \\ 3 & -4 & -1 & 1 \end{array} \right]$$

$$R_2 = (-3)r_1 + r_2:$$

$$R_3 = (-3)r_1 + r_3:$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 3 & 2 & 16 \\ 0 & 2 & 2 & 16 \end{array} \right]$$

$$\underline{r_2 = (-1)r_3 + r_2}:$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & \textcircled{1} & 0 & 0 \\ 0 & 2 & 2 & 16 \end{array} \right]$$

$$\underline{R_3 = (-2)r_2 + r_3}:$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 16 \end{array} \right]$$

$$\underline{R_3 = \left(\frac{1}{2}\right)r_3}:$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 8 \end{array} \right]$$

$$\therefore z = 8$$

$$y = 0$$

$$x - 2y - z = -5$$

$$x - 2(0) - 8 = -5$$

$$x = 3$$

$$\therefore x = 3, y = 0, z = 8$$