(1) [6] Suppose

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, \quad A = \{0, 1, 5, 7\}, \quad B = \{0, 2, 3, 5, 8\}, \quad C = \{5, 6, 8, 9\}$$

(i) Determine $A \cup (B \cap A)$

$$B \cap A = \{0, 5\}$$

$$\therefore A \cup (B \cap A) = \{0, 1, 5, 7\}$$

$$\boxed{\text{Notice: since } (B \cap A) \subset A, \quad A \cup (B \cap A) = A.}$$

(ii) Determine $(C \cap A) \cap (\overline{A})$

$$C \cap A = \{5\}, \quad \overline{A} = \{2, 3, 4, 6, 8, 9\}$$

$$\therefore (C \cap A) \cap (\overline{A}) = \varnothing$$

$$\boxed{\text{Notice: since } C \cap A \subset A, \quad (C \cap A) \cap (\overline{A}) = \varnothing.}$$

(iii) Determine $\overline{A} \cap \overline{B}$

$$\overline{A} = \{2, 3, 4, 6, 8, 9\}, \quad \overline{B} = \{1, 4, 6, 7, 9\}$$

$$\therefore \overline{A} \cap \overline{B} = \{4, 6, 9\}$$

(2) [4] The user names for a computer network consist of four uppercase letters selected from $\{A, B, C, \ldots, Z\}$ followed by four digits selected from $\{0, 1, 2, \ldots, 9\}$. How many possible user names have no matching adjacent letters or digits?

User names have form $\underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad} \mid \underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}$

$\underbrace{\qquad\qquad}_{\text{letters}}$ $\underbrace{\qquad\qquad}_{\text{digits.}}$

For the letters, there are 26 choices for the first letter, and 25 choices for each of the next three. Similarly, for the digits, there are 10 choices for the first digit and 9 for each of the following three. So in total there are $(26)(25)^3 (10)(9)^3 = 2,961,562,500$ possible user names.

(3) [5] 250 fitness club members are surveyed to determine the type of equipment they use regularly. 170 replied that they use an exercise bike regularly, 119 said they use a stair stepper machine regularly, and 67 said they use both regularly. How many of those surveyed use neither of the machines regularly?
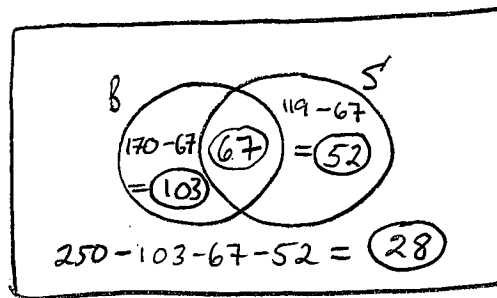
$B:$ bike

$S:$ stair machine.

$\therefore$ 28 use neither of the machines regularly.