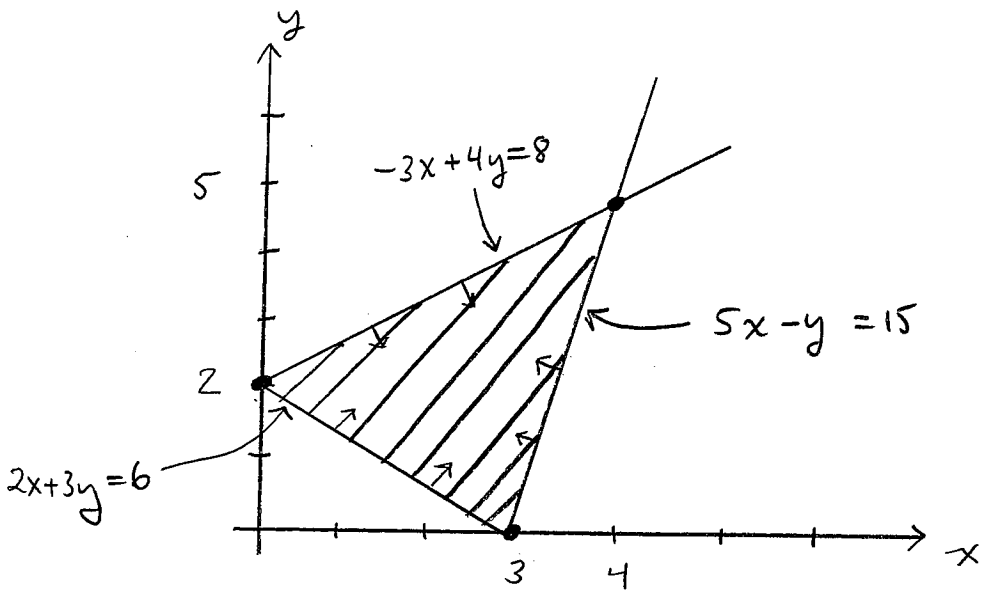
(1) [15] Maximize

$$z = 7x + 6y$$

subject to

$$\begin{aligned} 2x + 3y &\geq 6 \\ -3x + 4y &\leq 8 \\ 5x - y &\leq 15 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Inequality	Line	test point	test result
$2x+3y \geq 6$	$2x+3y=6$	$(0,0)$	$2(0)+3(0) \geq 6$: false
$-3x+4y \leq 8$	$-3x+4y=8$	$(0,0)$	$-3(0)+4(0) \leq 8$: true
$5x-y \leq 15$	$5x-y=15$	$(0,0)$	$5(0)-0 \leq 15$: true.



Corner Points: By inspection: $(0,2)$, $(3,0)$.

Solving: ① $5x-y=15$

② $-3x+4y=8$

① $\Rightarrow$ $y = 5x-15$

② $\Rightarrow$ $-3x+4(5x-15)=8$

$\Rightarrow$ $-3x+20x-60=8$

cont'd →

$17x = 68$

$x = 4$

$\therefore y = 5x - 15$

$= 5(4) - 15$

$= 5$

$\therefore (4,5)$ is a corner point.

Corner point	$z = 7x + 6y$
$(0,2)$	$z = 7(0) + 6(2) = 12$
$(3,0)$	$z = 7(3) + 6(0) = 21$
$(4,5)$	$z = 7(4) + 6(5) = 58$

$\therefore z$ has a maximum of 58 at $x = 4, y = 5$.