 Name: SOLUTIONS.

(1) [5] Let

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 \\ 5 & 1 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

Compute $4\mathbf{A} + 3(\mathbf{B} + \mathbf{C})$.

$$4A + 3(B+C) = 4\begin{bmatrix} 1 & -2 & 0 \\ 5 & 1 & 2 \end{bmatrix} + 3\left(\begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 4 & -8 & 0 \\ 20 & 4 & 8 \end{bmatrix} + 3\begin{bmatrix} -1 & -3 & 9 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -8 & 0 \\ 20 & 4 & 8 \end{bmatrix} + \begin{bmatrix} -3 & -9 & 27 \\ 6 & 9 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -17 & 27 \\ 26 & 13 & 20 \end{bmatrix}$$

(2) [5] Compute the following product:

$$\begin{bmatrix} 1 & -1 & 6 \\ 2 & 0 & -1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 1 \\ 5 & 4 \\ 11 & 7 \end{bmatrix}.$$

(3) [5] Let $\mathbf{A} = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$. Determine $\mathbf{A}^{-1}$. Clearly label all row operations used.

$$\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right]$$

$r_1 \leftrightarrow r_2$: $\left[\begin{array}{cc|cc} 3 & 7 & 0 & 1 \\ 2 & 5 & 1 & 0 \end{array}\right]$

$R_1 = (-1) r_2 + r_1$: $\left[\begin{array}{cc|cc} 1 & 2 & -1 & 1 \\ 2 & 5 & 1 & 0 \end{array}\right]$

$R_2 = (-2) r_1 + r_2$: $\left[\begin{array}{cc|cc} 1 & 2 & -1 & 1 \\ 0 & 1 & 3 & -2 \end{array}\right]$

$R_1 = (-2) r_2 + r_1$:

$$\left[\begin{array}{cc|cc} 1 & 0 & -7 & 5 \\ 0 & 1 & 3 & -2 \end{array}\right]$$

$\therefore A^{-1} = \begin{bmatrix} -7 & 5 \\ 3 & -2 \end{bmatrix}$

Check:

$AA^{-1} = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} -7 & 5 \\ 3 & -2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ☺