

(1) [4] Determine the market price if $S = 20p + 500$ and $D = 1000 - 30p$.

Solve $S = D$

$$20p + 500 = 1000 - 30p$$

$$50p = 500$$

$$p = \$10$$

(2) [4] Solve the following system of equations or say that it is inconsistent:

$$\begin{cases} \textcircled{1} & 3x - 6y = 2 \\ \textcircled{2} & 5x + 4y = 1 \end{cases}$$

new $\textcircled{1} = 5(\text{old } \textcircled{1})$:

new $\textcircled{2} = -3(\text{old } \textcircled{2})$:

$$\begin{cases} \textcircled{1} & 15x - 30y = 10 \\ \textcircled{2} & -15x - 12y = -3 \end{cases}$$

new $\textcircled{2} = \text{old } \textcircled{1} + \text{old } \textcircled{2}$:

$$\textcircled{1} \quad 15x - 30y = 10$$

$$\textcircled{2} \quad 0x - 42y = 7$$

$$\therefore y = -\frac{7}{42} = -\frac{1}{6}$$

$$15x - 30y = 10$$

$$15x - 30\left(-\frac{1}{6}\right) = 10$$

$$\begin{aligned} 15x &= 5 \\ x &= \frac{5}{15} = \frac{1}{3} \end{aligned}$$

$$\therefore (x, y) = \left(\frac{1}{3}, -\frac{1}{6}\right)$$

(3) [7] For a certain commodity the supply equation is

$$S = 2p + 5$$

At a price of \$1 there is a demand for 19 units of the commodity. Find the demand equation if the demand equation is linear and the market price is \$3.

When $p = 1$, $D = 19$, so $(1, 19)$ is a point on demand line.

When $p = 3$, $S = 2(3) + 5 = 11$, so $(3, 11)$ is a point on supply line. Since $p = 3$ is the market price, $(3, 11)$ is also on the demand line.

∴ Demand equation is line through $(1, 19)$ and $(3, 11)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{19 - 11}{1 - 3} = \frac{8}{-2} = -4$$

$$\therefore D - 19 = -4(p - 1)$$

$$D - 19 = -4p + 4$$

$$\boxed{D = -4p + 23}$$