Question 1 [10 points]: For this question you may state equations of lines using any of the standard forms.

(a)[4] Determine an equation of the line passing through the points $(-7, 3)$ and $(-2, -3)$.

(b)[3] Determine an equation of the vertical line through the point $(11, -11)$.

(c)[3] Determine an equation of the line which is parallel to the line $3x - 5y = 12$ and which has y-intercept $(0, 3)$.

Question 2 [10 points]:

(a)[5] A doughnut shop sells doughnuts for \$3.29 per dozen. The shop has fixed weekly costs of \$650 and it costs \$1.55 to make a dozen doughnuts. How many dozen doughnuts must be sold each week in order for the business to break even? Round your final answer to the nearest dozen.

(b)[5] A company has data indicating that when the price of a particular product is \$138 the quantity demanded is 72 while the quantity supplied is 96. At the market (or equilibrium) price of \$120 the quantity demanded increases to 88. Determine the supply equation for the product.

Question 3 [10 points]:

(a)[5] A local charity sells boxes of oranges and boxes of grapefruit to raise money. A box of oranges costs \$14 and a box of grapefruit costs \$16. The fundraising drive ended up raising \$7570 from the sale of 502 boxes of fruit. How many boxes of each type of fruit were sold?

(b)[5] An investor has \$12,000 to invest and two investments are available: one pays 4% simple interest per year, while the second riskier investment pays 6.5% simple interest per year. The investor has a goal of earning \$570 for the year. How much should be invested in each of the investments?

Question 4 [10 points]: Solve the following system of equations **using matrix reduction** (no credit will be given for using any other method). Use proper notation to clearly state the row operations used at each step and clearly state the final solution.

$$\begin{aligned} -x + 2y + z &= 1 \\ -5x + 8y + 2z &= 3 \\ 7x - 11y - 3z &= -2 \end{aligned}$$

Question 5 [10 points]: For this problem use the following matrices:

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 1 & 0 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

(a)[3] Compute $\mathbf{AB} - 4\mathbf{C}$.

(b)[4] Determine $\mathbf{C}^{-1}$.

(c)[3] Solve

$$\mathbf{C} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

for x and y. Your answer to part (b) should make this easy.

Question 6 [10 points]: Maximize $z = 4x + 6y$ subject to the constraints

$$\begin{aligned} 5x + 3y &\geq 15 \\ x + 2y &\leq 20 \\ 7x + 9y &\leq 105 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Keep your work organized: neatly draw any required graphs and clearly show your work when determining corner points. State a clear conclusion.

Question 7 [10 points]:

(a)[3] What effective rate of interest is equivalent to 5.5% compounded monthly? (State your final answer as a percentage rounded to two decimal places.)

(b)[3] A deposit made today has a value of \$10,000 at the end of 10 years. If the interest rate is 5.6% compounded quarterly what was the original deposit amount? (Round your final answer to two decimal places.)

(c)[4] An investment triples in 12 years. What rate of interest compounded semiannually will achieve this? (State your final answer as a percentage rounded to two decimal places.)

Question 8 [10 points]:

(a)[5] Smithers' 45^{th} birthday is on January 1 2012 and he has decided that it is time to get serious about retirement. He will deposit $20,000 on his upcoming birthday, and then make deposits of $750 at the end of each month for the next 15 years. All deposits will be made to a fund earning 7.5% interest compounded monthly. How much will be in the fund when he reaches age 60? (Round your final answer to the nearest dollar.)

(b)[5] A $300,000 bank loan will be repaid by making payments at the end of every month. One bank offers 4% interest compounded monthly with a 20 year repayment period, while another bank offers 5% compounded monthly with a 25 year repayment period. Which repayment option results in the lowest monthly payment?

Question 9 [10 points]:

(a)[2] A senior's club has 40 female and 25 male members. An 8 person delegation of 4 female and 4 male members is to be selected to attend a regional meeting. How many different delegations are possible?

(b)[2] A bank machine requires a six digit PIN code where the digits can be $0, 1, 2, \ldots, 9$. How many PIN codes have at least one repeated digit?

(c)[3] How many different ways can 3 red, 4 yellow and 5 blue bulbs be arranged in a string of Christmas tree lights with 12 sockets?

(d)[3] Suppose A and B are finite sets with the property that $n(A \cup B) = n(A \cap B)$. What is $n(A) - n(B)$? (Note: this question is slightly challenging: think about how the sets A and B must be related.)

Question 10 [10 points]:

(a)[3] The odds for event E are 1 to 2, and the odds for event F are 2 to 3. If $P(E \cap F) = 0$ determine the odds for $E \cup F$, that is, the odds for E or F.

(b)[2] Two balls are drawn, one after the other and without replacement, from a box containing 4 white, 3 green and 2 yellow balls. What is the probability that one ball is white and the other yellow?

(c)[2] Again, two balls are drawn, one after the other and without replacement, from a box containing 4 white, 3 green and 2 yellow balls. What is the probability that the first ball is white and the second is yellow?

(d)[3] A person pays \$3 to roll a pair of dice. If the person rolls a "double", meaning ⚀⚀ , ⚁⚁ , etc, then he wins \$12, otherwise he wins nothing. What is the expected net payoff of such a game?

SCRAP & Formulas

Name:

Simple interest formula:

$$A = P(1 + rt)$$

Compound interest formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Interest formula for continuously compounded interest:

$$A = Pe^{rt}$$

Amount of an annuity:

$$A = P\left[\frac{(1+i)^m - 1}{i}\right]$$

Present value of an annuity:

$$V = P\left[\frac{1-(1+i)^{-m}}{i}\right]$$