Question 1:

(a)[3] What is the effective rate of interest equivalent to 3.75% compounded monthly?

Solve $$1+R = \left(1+\frac{0.0375}{12}\right)^{12}$$

$$\therefore R = \left(1+\frac{0.0375}{12}\right)^{12} - 1$$

$$\boxed{R \doteq 3.82\%}$$

(b)[3] An investment pays 8% compounded semiannually. How much must be invested now in order to have \$120 in the investment account at the end of two and a half years?

$$A = P\left(1+\frac{r}{n}\right)^{nt} \quad \text{where } r=0.08,\ n=2,\ A=120,\ t=2.5$$

$$\therefore 120 = P\left(1+\frac{0.08}{2}\right)^{(2)(2.5)}$$

$$\therefore P = \frac{120}{(1+0.04)^5} = \boxed{\$98.63}$$

(c)[4] What rate of interest compounded quarterly will double an investment in 7 years?

Solve $$\cancel{P}\left(1+\frac{r}{n}\right)^{nt} = 2\cancel{P} \quad \text{where } n=4,\ t=7$$

$$\left(1+\frac{r}{4}\right)^{(4)(7)} = 2$$

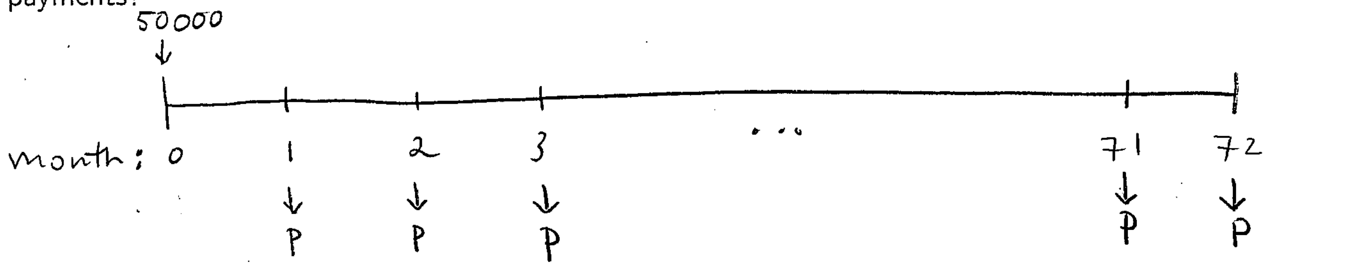
$$1+\frac{r}{4} = 2^{\frac{1}{28}}$$

$$r = 4\left(2^{\frac{1}{28}}-1\right)$$

$$r \doteq \boxed{10.03\%}$$

Question 2:

(a)[5] An inheritance worth \$50,000 on January 1 2012 is to be paid out in equal payments made at the end of each month for the next six years. The first payment will be made on January 31 2012. If the original \$50,000 is invested in a fund which pays 6% interest compounded monthly, how much are the monthly payments?

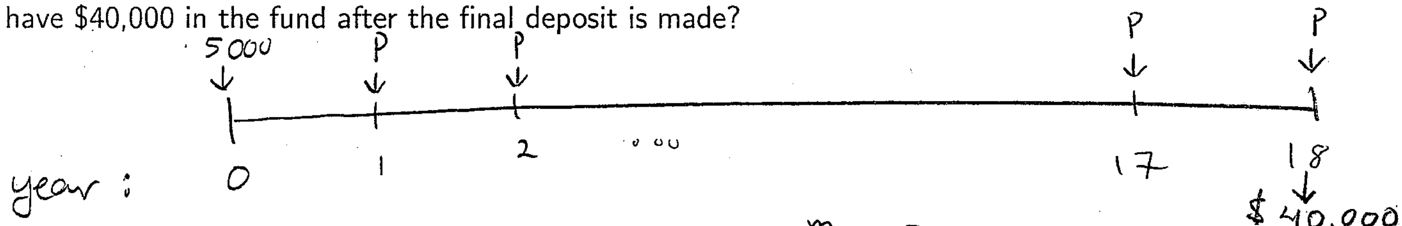


$$V = P\left[\frac{1-(1+i)^{-m}}{i}\right] \quad \text{where } V = 50000,\ i = \frac{0.06}{12} = 0.005,\ m = 72.$$

$$\therefore\ P = \frac{V}{\left[\frac{1-(1+i)^{-m}}{i}\right]}$$

$$= \frac{50000}{\left[\frac{1-(1.005)^{-72}}{0.005}\right]} \doteq \boxed{\$\,828.64}$$

(b)[5] The parents of a newborn wish to start saving for the child's university education. An investment fund which pays 5% compounded annually is to be used. The parents make an initial deposit of \$5000 on the day of the child's birth, and they will make deposits of $\$P$ on every subsequent birthday up to and including the day on which the child turns 18. How much should each of the deposits be if the goal is to have \$40,000 in the fund after the final deposit is made?



We need $\underbrace{5000\left(1+\frac{r}{n}\right)^{nt}}_{r=0.05,\ n=1,\ t=18} + \underbrace{P\left[\frac{(1+i)^m-1}{i}\right]}_{i=0.05,\ m=18} = 40{,}000$

$$\therefore\ 5000(1+0.05)^{18} + P\left[\frac{(1+0.05)^{18}-1}{0.05}\right] = 40{,}000$$

$$\therefore\ P = \frac{40\,000 - 5000(1.05)^{18}}{\left[\frac{(1.05)^{18}-1}{0.05}\right]} \doteq \boxed{\$\,994.12}$$

Question 3:

(a)[5] Let

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, \quad D = \{1, 3, 5, 7, 9\}, \quad E = \{0, 2, 4, 6, 8\} \quad , P = \{2, 3, 5, 7\}$$

Determine

(i) $D \cap (\overline{E} \cup \overline{P})$

$\overline{E} = \{1, 3, 5, 7, 9\}$

$\overline{P} = \{0, 1, 4, 6, 8, 9\}$

$\overline{E} \cup \overline{P} = \{0, 1, 3, 4, 5, 6, 7, 8, 9\}$

$\therefore\ D \cap (\overline{E} \cup \overline{P}) = \{1, 3, 5, 7, 9\}$

(ii) $(D \cap P) \cup \overline{(E \cap P)}$

$= \{3, 5, 7\} \cup \overline{\{2\}}$

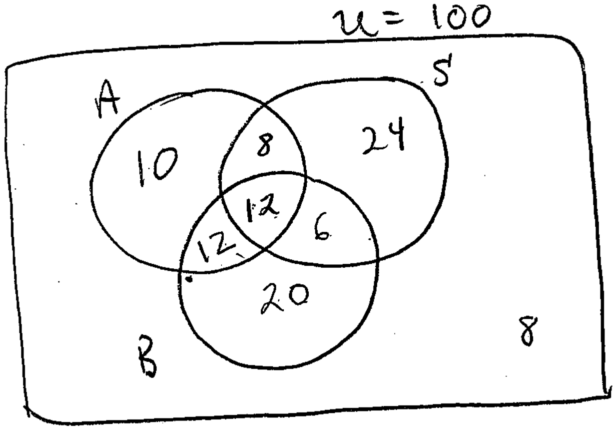
$= \{3, 5, 7\} \cup \{0, 1, 3, 4, 5, 6, 7, 8, 9\}$

$= \{0, 1, 3, 4, 5, 6, 7, 8, 9\}$

(b)[5] 100 new VIU first year students are surveyed to determine which courses, if any, they plan to take in arts, science and business. Of the 100 responses, 50 say they plan to take some science courses, and 50 say they plan to take some business courses. 20 plan to take both arts and science, 18 both science and business, and 24 both arts and business. 12 students say they will take all three types of courses, while 8 say they will take none of the three. How many students say they will take some arts courses?

A: arts, S: science, B: business

$u = 100$



$\therefore$ 42 say they will take some arts courses.

Question 4: For this question suppose the first 12 letters of the alphabet are used to construct 6-letter codes.

(a)[3] How many codes are possible if repetition is allowed but the first letter cannot be 'a'?

___, ___, ___, ___, ___, ___

11 choices

12 choices

$\therefore$ number of possible codes is $(11)(12)^5 = \boxed{2{,}737{,}152}$

(b)[3] How many codes are possible if repetition is not allowed and each code must begin or end with the letter 'a'?

number beginning with a: ___, ___, ___, ___, ___, ___

1 choice, 11 choices, 10, 9, 8, 7

$\therefore$ number beginning with a is $(11)(10)(9)(8)(7) = 55\,440$

Similarly, number ending with a is $55\,440$.

$\therefore$ number of codes possible is $(55\,440)(2) = \boxed{110{,}880.}$

(c)[4] Suppose codes are created at random and repetition is allowed. What is the probability that a code consists of the the same letter repeated six times?

S = set of all possible codes ; $n(S) = (12)^6$

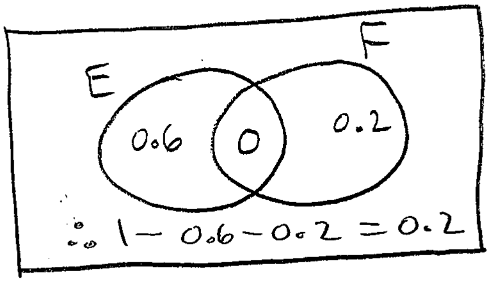
E = codes consisting of same letter repeated six times;

$n(E) = 12$

$\therefore P(E) = \dfrac{12}{(12)^6} = \dfrac{1}{12^5} = \boxed{\dfrac{1}{248{,}832} \doteq 0.000004}$

Question 5:

(a)[3] Suppose $P(E) = 0.6$ and $P(F) = 0.2$. What is the probability that neither E nor F occurs if it is known that the two events share no outcomes in common?



$$\therefore P\big(\underbrace{\overline{E \cup F}}_{\text{not } E \text{ or } F}\big) = 0.2$$

(b)[3] In a family of 5 people, what is the probability that at least two of them have birthdays in the same month?

E = at least two have same birthday month.

$\overline{E}$ = all have different birthday months.

$$P(E) = 1 - P(\overline{E})$$

$$= 1 - \frac{(12)(11)(10)(9)(8)}{(12)^5}$$

$$\doteq \boxed{0.62}$$

(c)[4] A European roulette wheel has 18 red slots, 18 black slots and 1 green slot. A marble is tossed onto the rotating wheel and it randomly falls into one of the 37 slots. A player pays \$1 to predict whether the ball will fall into a red or black slot. If correct, the player receives \$2, and receives nothing if incorrect. What is the expected net payoff of such a game?

A_1 = "guess is correct"; $P(A_1) = \frac{18}{37}$, $m_1 = 2 - 1 = 1$

A_2 = "guess is incorrect"; $P(A_2) = \frac{19}{37}$, $m_2 = -1$

$$E = m_1 P(A_1) + m_2 P(A_2)$$

$$= (1)\left(\frac{18}{37}\right) + (-1)\left(\frac{19}{37}\right)$$

$$\doteq \boxed{\$-0.027}$$

Question 6: BONUS

(a)[2] What is the effective rate of interest equivalent to 7% interest compounded continuously?

Solve $1 + R = e^{(0.07)(1)}$

$\therefore R = e^{0.07} - 1$

$R \doteq \boxed{7.25\%}$

(b)[3] How long will it take money invested at 9% compounded annually to double?

Solve $\cancel{P}\left(1 + \frac{0.09}{1}\right)^{(1)(t)} = 2\cancel{P}$ for t

$(1.09)^t = 2$

$\log_{10} (1.09)^t = \log_{10} 2$

$t \cdot \log_{10}(1.09) = \log_{10} 2$

$t = \frac{\log_{10} 2}{\log_{10}(1.09)} \doteq \boxed{8 \text{ yrs}}$