Question 1:

(a)[2 points] Determine the slope and y-intercept of the line $3x - 2y = 5$.

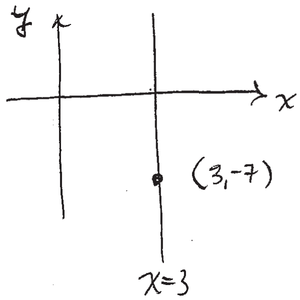
$$3x - 2y = 5$$

$$2y = 3x - 5$$

$$y = \frac{3}{2}x - \frac{5}{2}$$

$\therefore\ m = \frac{3}{2}$, y-intercept $(0, -\frac{5}{2})$

(b)[2 points] Determine an equation of the line through $(3, -7)$ with slope undefined.



$$\boxed{x = 3}$$

(c)[3 points] Determine an equation of the line through the points $(7, 1)$ and $(3, -7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-7)}{7-3} = \frac{8}{4} = 2$$

$$\boxed{\therefore\ y - 1 = 2(x - 7)}$$

or $y = 2x - 13$

(d)[3 points] Determine an equation of the line that is parallel to the line $x - 2y = -3$ and which passes through the point $(-3, 8)$.

$$x - 2y = -3$$

$$2y = x + 3$$

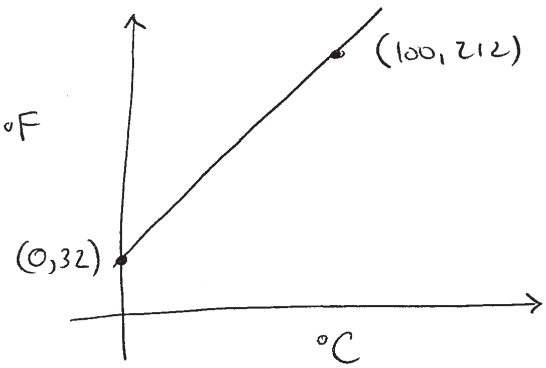
$$y = \frac{1}{2}x + \frac{3}{2}$$

$\therefore\ m = \frac{1}{2}$

$\therefore\ \boxed{y - 8 = \frac{1}{2}(x + 3)}$ or $y = \frac{1}{2}x + \frac{19}{2}$

Question 2:

(a)[5 points] The relationship between degrees Celsius (°C) and degrees Fahrenheit (°F) is linear. Find a linear relation (that is, the equation of the line) relating °C and °F if 0°C corresponds to 32°F and 100°C corresponds to 212°F, and then use the relation you found to convert 68°F to °C .



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

$$\therefore \quad F - 32 = \frac{9}{5}(C-0)$$

$$\boxed{F = \frac{9}{5}C + 32}$$

If $F = 68$,

$$68 = \frac{9}{5}C + 32$$

$$\frac{9}{5}C = 36$$

$$C = \left(\frac{5}{9}\right)(36)$$

$$\boxed{C = 20}$$

(b)[5 points] Determine whether the following lines are parallel, coincident or intersecting:

$$L: \; 4x - 2y = -7$$
$$M: \; -2x + y = -1$$

$L:\quad 4x - 2y = -7$

$2y = 4x + 7$

$y = 2x + \frac{7}{2}$

$M:\quad -2x + y = -1$

$y = 2x - 1$

Since slopes are equal but y-intercepts differ, lines are $\boxed{\text{parallel}}$.

Question 3:

(a)[5 points] Determine the point of intersection of the lines

$$L: \; 4x+3y=2$$
$$M: \; 2x-y=1$$

Using M: $y = 2x-1$

Sub. ↗ into L: $4x + 3(2x-1) = 2$

$4x + 6x - 3 = 2$

$10x = 5$

$x = \frac{1}{2}$

$\therefore y = 2(\frac{1}{2}) - 1 = 0$

$\therefore \boxed{(\frac{1}{2}, 0)}$

(b)[5 points] A person has 20 coins which total \$1.65. If the coins consist of nickels (5¢ pieces) and dimes (10¢ pieces), how many of each type of coin does the person have? Clearly define your variables and state a clear conclusion.

Let x = no. of nickels
y = no. of dimes.

① $x + y = 20$

② $5x + 10y = 165$

① $\Rightarrow$ $y = 20 - x$

② $\Rightarrow$ $5x + 10(20-x) = 165$

$5x + 200 - 10x = 165$

$5x = 35$

$x = 7$

$\therefore y = 20 - 7 = 13$

∴ The person has 7 nickels and 13 dimes.

Question 4:

(a)[5 points] A certain product has supply equation $S = 40p + 300$ and a market price of \$30. Each \$4 increase in price reduces demand by 100 units. At what price does demand drop to 1175 units?

At $p = 30$, $D = S = 40(30) + 300 = 1500$.

$\therefore$ $(30, 1500)$ is on D-line.

If $p = 30+4$, $D = 1500 - 100$; so $(34, 1400)$ is also on D-line.

$\therefore$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1500-1400}{30-34} = -25$

$\therefore$ $D - 1500 = -25(p - 30)$

When $D = 1175$: $1175 - 1500 = -25(p-30)$

$$-325 = -25p + 750$$

$$p = \frac{1075}{25} = 43$$

$\boxed{\therefore \text{ Demand drops to 1175 at } p = \$43}$

(b)[5 points] A ferry service has different fare options. One option is to pay a one-time fee of \$52 to join their frequent traveller club and then pay \$15 for every trip on the ferry. A second option is to simply pay a \$19 fare for each trip without joining the frequent traveller club. How many trips are required for both options to be equivalent in terms of cost?

Let x = the required number of trips.

We require

(cost if member of club) = (regular cost)

$$52 + 15x = 19x$$

$$52 = 4x$$

$$x = 13$$

$\boxed{\therefore \text{ 13 trips are required.}}$

Question 5 [10]: Solve the following system of equations **using matrix reduction** (no credit will be given for using any other method). Use proper notation to clearly state the row operations used at each step and clearly state the final solution.

$$\begin{aligned} 3x - 3y - z &= 1 \\ -x + 2y + z &= 5 \\ 3x - 4y - z &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & -3 & -1 & 1 \\ -1 & 2 & 1 & 5 \\ 3 & -4 & -1 & 1 \end{array}\right]$$

$\underline{r_1 \leftrightarrow r_2}$:

$$\left[\begin{array}{ccc|c} -1 & 2 & 1 & 5 \\ 3 & -3 & -1 & 1 \\ 3 & -4 & -1 & 1 \end{array}\right]$$

$\underline{R_1 = (-1)r_1}$:

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 3 & -3 & -1 & 1 \\ 3 & -4 & -1 & 1 \end{array}\right]$$

$R_2 = (-3)r_1 + r_2$:

$R_3 = (-3)r_1 + r_3$:

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 3 & 2 & 16 \\ 0 & 2 & 2 & 16 \end{array}\right]$$

$\underline{R_2 = (-1)r_3 + r_2}$:

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 16 \end{array}\right]$$

$\underline{R_3 = (-2)r_2 + r_3}$:

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 16 \end{array}\right]$$

$\underline{R_3 = \left(\tfrac{1}{2}\right) r_3}$:

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

$\therefore\ z = 8$

$y = 0$

$x - 2y - z = -5$

$x - 2(0) - 8 = -5$

$x = 3$

$$\boxed{\therefore\ x = 3,\ y = 0,\ z = 8}$$