Additional Problems for Assignment 4

1. Suppose $\sum\limits_{n=1}^{\infty} x_n$ is conditionally convergent, and define

$$a_n = \max\{x_n, 0\}$$
$$b_n = \min\{x_n, 0\}$$

   Show that $\sum\limits_{n=1}^{\infty} a_n$ and $\sum\limits_{n=1}^{\infty} b_n$ are both divergent.

2. Find an infinite sequence of closed discs $D_1, D_2, D_3, \ldots$ in the plane with centres $c_1, c_2, c_3, \ldots$, respectively, such that

   (a) Every line in the plane intersects at least one of the D_i, and

   (b) The sum of the areas of the D_i is finite.

   (hint: think about the harmonic series, and focus on the axes.)

3. Determine $\lim\limits_{x \to 2} \dfrac{1}{x}$. Prove your result using the $\epsilon\delta$ definition of the limit.