Question 1:

(a)[8] Use matrix reduction to solve the following system of equations:

$$6x + y = 8$$
$$x - 3y = -5$$
$$2x + y = 2$$

$$\left[\begin{array}{cc|c} 6 & 1 & 8 \\ 1 & -3 & -5 \\ 2 & 1 & 2 \end{array}\right]$$

$r_1 \leftrightarrow r_2$:
$$\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 6 & 1 & 8 \\ 2 & 1 & 2 \end{array}\right]$$

$R_2 = (-6)r_1 + r_2$:

$R_3 = (-2)r_1 + r_3$:
$$\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 19 & 38 \\ 0 & 7 & 12 \end{array}\right]$$

$R_2 = \frac{1}{19} r_2$:
$$\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 7 & 12 \end{array}\right]$$

$\therefore$ The system has no solution

$R_3 = (-7)r_2 + r_3$:
$$\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{array}\right]$$

(b)[2] Is the system of equations in (a) consistent or inconsistent?

System is inconsistent

Question 2: For this problem use the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 3 & -4 \\ 1 & 4 \\ 5 & -2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

(a)[4] Compute $(3\mathbf{A} - 4\mathbf{C})\mathbf{D}$

$$3A - 4C = \begin{bmatrix} 3 & 0 \\ 6 & 12 \\ -3 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -16 \\ 4 & 16 \\ 20 & -8 \end{bmatrix} = \begin{bmatrix} -9 & 16 \\ 2 & -4 \\ -23 & 5 \end{bmatrix}$$

$$(3A - 4C)D = \underbrace{\begin{bmatrix} -9 & 16 \\ 2 & -4 \\ -23 & 14 \end{bmatrix}}_{3\times 2} \underbrace{\begin{bmatrix} -6 \\ 1 \end{bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} 70 \\ -16 \\ 152 \end{bmatrix}}_{3\times 1}$$

(b)[4] Compute $\mathbf{AB} - 3\mathbf{I_3}$

$$\begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & 0 \\ 12 & -2 & -8 \\ -2 & 5 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 0 \\ 12 & -5 & -8 \\ -2 & 5 & -7 \end{bmatrix}.$$

(c)[2] Suppose there is some matrix $\mathbf{P}$ such that the product $\mathbf{BPA}$ is defined. What must be the dimension of the matrix $\mathbf{P}$?

$$\begin{matrix} B & P & A \\ 2\times 3 & 3\times 3 & 3\times 2 \end{matrix}$$

$\boxed{\therefore\ P \text{ must have dimension } 3\times 3.}$

Question 3:

(a)[7] Determine $\mathbf{A}^{-1}$ where $\mathbf{A}$ is the matrix

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = r_1 + r_2$:

$R_3 = (-1)r_1 + r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$R_1 = r_2 + r_1$:

$R_3 = (-1)r_2 + r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & -1 & -2 & -1 & 1 \end{array}\right]$$

$R_3 = (-1)r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right]$$

$R_1 = (-3)r_3 + r_1$:

$R_2 = (-3)r_3 + r_2$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -2 & 3 \\ 0 & 1 & 0 & -5 & -2 & 3 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right]$$

$$\boxed{\therefore\ A^{-1} = \begin{bmatrix} -4 & -2 & 3 \\ -5 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix}}$$

(b)[3] Use your result in part (a) to solve the following system of equations:

$$\begin{aligned} x - y &= -3 \\ -x + 2y + 3z &= -1 \\ x + 2z &= 7 \end{aligned}$$

System is
$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 7 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 & -2 & 3 \\ -5 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ -1 \\ 7 \end{bmatrix} = \begin{bmatrix} 35 \\ 38 \\ -14 \end{bmatrix}$$

$$\boxed{\therefore\ x = 35,\ y = 38,\ z = -14}$$

Question 4 [10 points]: A certain animal requires at least 30 grams of protein and 20 grams of fat each feeding. Two foods are available: food A costs \$0.18 per unit, and each unit supplies 2 grams of protein and 4 grams of fat. Food B costs \$0.12 per unit, and each unit provides 6 grams of protein and 2 grams of fat. At least 2 units of food B must be used each feeding. How many units of foods A and B should be used each feeding to minimize cost?

Graph paper is provided on the next page. Carefully set up the problem, neatly sketch any required graphs and <u>state a clear conclusion</u>.

Let x = units of Food A
y = units of Food B
z = cost per feeding.

minimize $z = 0.18x + 0.12y$

subject to
$2x + 6y \geq 30$ } protein
$4x + 2y \geq 20$ } fat
$y \geq 2$
$x \geq 0$
$y \geq 0$.

See next page for graph.

<u>Corner points:</u> • By inspection: $(0,10)$

• Intersection of ① $4x+2y=20$, ② $2x+6y=30$

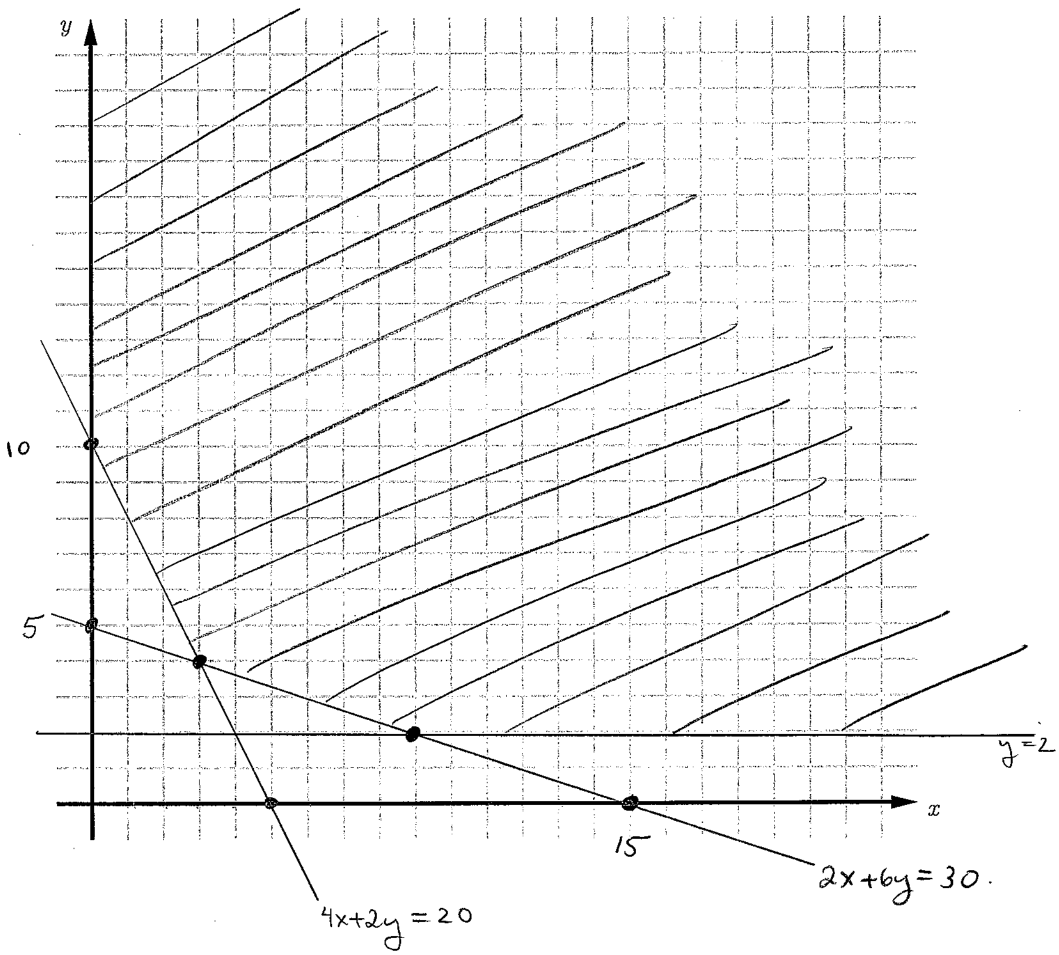
②×2:
$$\begin{array}{r} 4x+2y = 20 \\ -\ \ 4x+12y = 60 \\ \hline -10y = -40 \end{array}$$
$y = 4$
$\therefore 2x+6y = 30$
$2x+6(4) = 30$
$x = 3$
$\therefore (3,4)$

• Intersection of $2x+6y=30$, $y=2$

$\therefore 2x+6(2)=30$, $x=9$ } $\therefore (9,2)$.

Question 4 (continued)

corner pt	$z = 0.18x + 0.12y$
$(0, 10)$	$z = (0.18)(0) + (0.12)(10) = \$1.20$
$(3, 4)$	$z = (0.18)(3) + (0.12)(4) = \$1.02$ ← minimum
$(9, 2)$	$z = (0.18)(9) + (0.12)(2) = \$1.86$



∴ 3 units of Food A and 4 units of Food B should be used to minimize cost.