Question 1:

(a)[3] What amount must be invested at 8% compounded quarterly to have \$1500 in three years time?

$A = 1500$
$r = 0.08$
$n = 4$
$t = 3$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$\therefore P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$

$$= \frac{1500}{\left(1+\frac{0.08}{4}\right)^{(4)(3)}}$$

$$P \approx \$1182.74$$

(b)[3] What rate of interest compounded monthly is equivalent to an effective rate of 5.5% ?

$n = 12$

$R = 0.055$

Solve $1 + 0.055 = \left(1+\frac{r}{12}\right)^{12}$

$$\therefore (1.055)^{\frac{1}{12}} = 1+\frac{r}{12}$$

$$(1.055)^{\frac{1}{12}} - 1 = \frac{r}{12}$$

$$\therefore r = \left[(1.055)^{\frac{1}{12}} - 1\right]12$$

$$\therefore r \approx 0.0537 = 5.37\%$$

(c)[4] One person invests \$1000 at 6% simple interest. A second person invests \$$P$ at 7% compounded annually. At the end of two years both investments have the same value. What is P ?

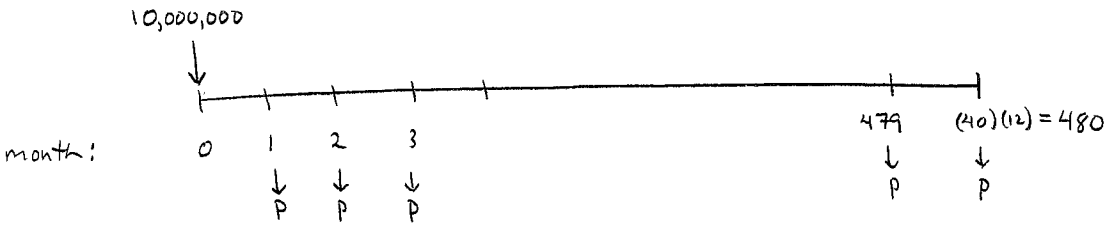
Solve $1000\left(1+(0.06)(2)\right) = P\left(1+0.07\right)^2$

$$\therefore P = \frac{1000\left[1.12\right]}{(1.07)^2}$$

$$P \approx \$978.25$$

Question 2:

(a)[5] A person wins \$10,000,000 in the lottery and decides to invest all of the money in an investment which earns 4% interest compounded monthly. The person plans to withdraw equal amounts at end of each month for the next 40 years. How much can be withdrawn each month so that no money is left in the investment fund after the final withdrawal is made?



$V = 10{,}000{,}000$

$r = 0.04,$

$n = 12$

$i = \frac{r}{n} = \frac{0.04}{12}$

$m = nt = 480.$

$\therefore V = P\left[\frac{1-(1+i)^{-m}}{i}\right]$

$\therefore P = \frac{V}{\left[\frac{1-(1+i)^{-m}}{i}\right]}$

$$P = \frac{10{,}000{,}000}{\left[\frac{1-\left(1+\frac{0.04}{12}\right)^{-480}}{\left(\frac{0.04}{12}\right)}\right]}$$

$$\boxed{P \approx \$41{,}793.85}$$

(b)[5] Which is a better investment interest rate: 5.5% compounded semiannually or 5.2% compounded monthly?

Let R_1 be the effective rate corresponding to 5.5% compounded semiannually.

$\therefore R_1 = \left(1+\frac{0.055}{2}\right)^2 - 1 \approx 5.5756\,\%$

Let R_2 be the effective rate corresponding to 5.2% compounded monthly.

$\therefore R_2 = \left(1+\frac{0.052}{12}\right)^{12} - 1 \approx 5.3257\,\%.$

Since $R_1 > R_2$, the first investment is more favourable.

Question 3:

(a)[5] Ned and Maude need to save money for the their sons' university education. They open a savings fund with an initial \$2000 deposit, and they plan to make regular equal deposits of \$$P$ at the end of each month for the next 15 years. At the end of the 15 years they want the total value of the fund to be \$120,000. Determine P if the fund pays 7.2% compounded monthly.

$r = 0.072$
$n = 12$
$t = 15$
$A = 120{,}000$
$m = nt = 180$
$i = \frac{r}{n} =$

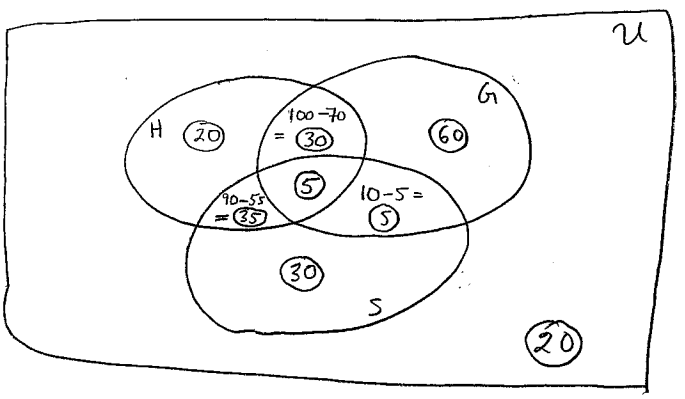
$\therefore$ We have

$$2000\left(1+\frac{0.072}{12}\right)^{(15)(12)} + P\left[\frac{(1+0.006)^{180}-1}{0.006}\right] = 120{,}000$$

$$\therefore P = \frac{120{,}000 - 2000(1.006)^{180}}{\left[\frac{(1.006)^{180}-1}{0.006}\right]}$$

$$P \approx \$353.86$$

(b)[5] Of the cars sold during the month of July, 90 had heated seats, 100 had GPS, and 75 had satellite radio. Five cars had all three of these extras. Twenty cars had none of these extras. Twenty cars had only heated seats; 60 cars had only GPS; and 30 cars had only satellite radio. Ten cars had both GPS and satellite radio. How many cars were sold in July?



H: Heated Seats
G: GPS
S: satellite radio

$\therefore$ Total number of cars is

$$20+30+5+35+60+5+30+20 = 205$$

Question 4:

(a)[3] Suppose A is a set containing 12 elements and B is a set containing 9 elements. If A and B share 4 elements in common, how many elements are in $A \cup B$?

$n(A) = 12$, $n(B) = 9$, $n(A \cap B) = 4$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 12 + 9 - 4$

$= \boxed{17}$

(b)[3] In how many different ways can a president, vice-president and secretary be selected from a group of 10 students?

choices : $\frac{10}{\text{Pres.}}, \frac{9}{\text{VP.}}, \frac{8}{\text{Sec.}}$

$\therefore$ number of ways is $P(10,3) = (10)(9)(8)$

$= \boxed{720}$

(c)[4] A restaurant menu lists 5 different vegetable and 3 different meat choices. Customers build their own combination plates by selecting 3 servings of vegetables and 2 servings of meat. How many different combination plates are possible if

(i) repetition of vegetable or meat selections is not allowed?

$C(5,3)\,C(3,2) = \frac{5!}{2!\,3!} \cdot \frac{3!}{1!\,2!}$

$= \boxed{30}$ different combination plates

(ii) repetition of vegetable or meat selections is allowed?

$(5^3)(3^2) = \boxed{1125}$ different combination plates.

Question 5:

(a)[3] How many different 11 letter codes can be formed using the letters 'MATHEMATICS'?

2 M's
2 A's
2 T's.

∴ there are $\frac{11!}{2!2!2!} = 4,989,600$ different codes.

(b)[3] A box contains 30 balls numbered 1 to 30. There are 8 red balls and the other 22 are green. Of all possible groups of 4 balls selected without replacement, how many contain exactly one red ball?

We have $C(8,1)$ ways of selecting a red ball,
$C(22,3)$ ways of selecting the 3 green balls.

∴ by the multiplication principle, there are

$$C(8,1)\,C(22,3) = \frac{8!}{7!\,1!} \cdot \frac{22!}{19!\,3!}$$

$$= \boxed{12,320}$$ groups of 4 containing exactly 1 red ball.

(c)[4] Again with the same box of balls as in part (b), of all possible groups of 4 balls selected without replacement, how many contain at least one red ball?

$$\begin{pmatrix}\#\text{ groups with}\\ \text{at least one}\\ \text{red}\end{pmatrix} = \begin{pmatrix}\#\text{ of possible}\\ \text{groups of 4}\end{pmatrix} - \begin{pmatrix}\#\text{ groups}\\ \text{containing no}\\ \text{red balls}\end{pmatrix}$$

$$= C(30,4) - C(22,4)$$

$$= \frac{30!}{26!\,4!} - \frac{22!}{18!\,4!}$$

$$= \boxed{20,090}$$