Question 1:

(a)[8] Use matrix reduction to solve the following system of equations:

$$6x + y = 8$$
$$x - 3y = -5$$
$$2x + y = 4$$

$$\left[\begin{array}{cc|c} 6 & 1 & 8 \\ 1 & -3 & -5 \\ 2 & 1 & 4 \end{array}\right]$$

$r_1 \leftrightarrow r_2$: $\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 6 & 1 & 8 \\ 2 & 1 & 4 \end{array}\right]$

$R_2 = (-6) r_1 + r_2$:
$R_3 = (-2) r_1 + r_3$:

$$\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 19 & 38 \\ 0 & 7 & 14 \end{array}\right]$$

$R_2 = \frac{1}{19} r_2$: $\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 7 & 14 \end{array}\right]$

$R_3 = (-7) r_2 + r_3$:

$$\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$\therefore \quad y = 2$

$x = -5 + 3y$
$= -5 + 3(2)$
$= 1$

$\boxed{\therefore x = 1, \ y = 2}$

(b)[2] Is the system of equations in (a) consistent or inconsistent?

System is consistent.

Question 2: For this problem use the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 3 & -4 \\ 1 & 4 \\ 5 & -2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

(a)[4] Compute $(3\mathbf{A} - 4\mathbf{C})\mathbf{D}$

$$3A - 4C = \begin{bmatrix} 3 & 0 \\ 6 & 12 \\ -3 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -16 \\ 4 & 16 \\ 20 & -8 \end{bmatrix} = \begin{bmatrix} -9 & 16 \\ 2 & -4 \\ -23 & 5 \end{bmatrix}$$

$$(3A - 4C)D = \begin{bmatrix} -9 & 16 \\ 2 & -4 \\ -23 & 14 \end{bmatrix} \begin{bmatrix} -6 \\ 1 \end{bmatrix} = \begin{bmatrix} 70 \\ -16 \\ 152 \end{bmatrix}$$

(b)[4] Compute $\mathbf{AB} - 3\mathbf{I}_3$

$$\begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & 0 \\ 12 & -2 & -8 \\ -2 & 5 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 0 \\ 12 & -5 & -8 \\ -2 & 5 & -7 \end{bmatrix}.$$

(c)[2] Suppose there is some matrix $\mathbf{P}$ such that the product $\mathbf{APB}$ is defined. What must be the dimension of the matrix $\mathbf{P}$?

$$A \quad P \quad B$$

$$3\times 2 \quad 2\times 2 \quad 2\times 3$$

$\therefore$ P must have dimension 2×2.

Question 3:

(a)[7] Determine $\mathbf{A}^{-1}$ where $\mathbf{A}$ is the matrix

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = r_1 + r_2$:

$R_3 = (-1) r_1 + r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$R_1 = r_2 + r_1$:

$R_3 = (-1) r_2 + r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & -1 & -2 & -1 & 1 \end{array}\right]$$

$R_3 = (-1) r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right]$$

$R_1 = (-3) r_3 + r_1$:

$R_2 = (-3) r_3 + r_2$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -2 & 3 \\ 0 & 1 & 0 & -5 & -2 & 3 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -4 & -2 & 3 \\ -5 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix}$$

(b)[3] Use your result in part (a) to solve the following system of equations:

$$\begin{aligned} x - y &= -1 \\ -x + 2y + 3z &= -3 \\ x + 2z &= 5 \end{aligned}$$

System is $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 & -2 & 3 \\ -5 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 25 \\ 26 \\ -10 \end{bmatrix}$$

$\therefore x = 25, \quad y = 26, \quad z = -10$

Question 4 [10 points]: A certain animal requires at least 30 grams of protein and 20 grams of fat each feeding. Two foods are available: food A costs \$0.12 per unit, and each unit provides 6 grams of protein and 2 grams of fat. Food B costs \$0.18 per unit, and each unit supplies 2 grams of protein and 4 grams of fat. At least 2 units of food A must be used each feeding. How many units of foods A and B should be used each feeding to minimize cost?

Graph paper is provided on the next page. Carefully set up the problem, neatly sketch any required graphs and state a clear conclusion.

Let x = units of Food A
y = units of Food B
z = cost per feeding.

minimize : $z = 0.12x + 0.18y$

subject to :
$6x + 2y \geq 30$ } protein
$2x + 4y \geq 20$ } fat

$x \geq 2$
$x \geq 0$
$y \geq 0$.

see next page for graph.

Corner Points:
- By inspection: $(10, 0)$
- Intersection of $6x+2y=30$ ①
  $2x+4y=20$ ②.

②×3 :
$$\begin{aligned} 6x+2y &= 30 \\ -\quad 6x+12y &= 60 \\ \hline -10y &= -30 \\ y &= 3 \end{aligned}$$

$\therefore 2x+4(3) = 20$
$2x = 8$
$x = 4$
$\therefore (4,3)$.

- Intersection of $x=2$
  $6x+2y=30$

$\therefore 6(2)+2y = 30$
$\therefore y = \frac{30-12}{2} = 9$
$\therefore (2,9)$.

Question 4 (continued)

Corner pt	$z = 0.12x + 0.18y$
$(10,0)$	$z = (0.12)(10) + (0.18)(0) = \$1.20$
$(4,3)$	$z = (0.12)(4) + (0.18)(3) = \$1.02$ ← min.
$(2,9)$	$z = (0.12)(2) + (0.18)(9) = \$1.86$.

$\therefore$ 4 units of Food A and 3 units of Food B should be used to minimize cost.