Question 1:

(a)[3] Determine an equation of the line through the points $(-2, 3)$ and $(-4, -7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-3}{-4-(-2)} = \frac{-10}{-2} = 5$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 3 = 5(x+2)}$$

or $\boxed{y = 5x + 13}$

(b)[3] Determine whether the following lines are intersecting, parallel, or coincident:

$$L: \quad 2x + 3y = -5$$
$$M: \quad 4x + 6y = -10$$

$L:$ $3y = -2x - 5$

$y = \frac{-2}{3}x - \frac{5}{3}$

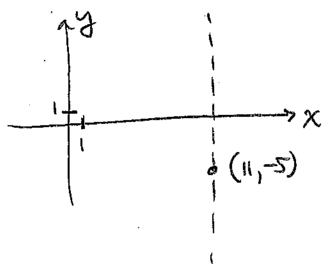
$M:$ $6y = -4x - 10$

$y = -\frac{4}{6}x - \frac{10}{6}$

$y = -\frac{2}{3}x - \frac{5}{3}$

Since slopes and intercepts are the same, lines are $\boxed{\text{coincident.}}$

(c)[2] Determine the equation of the vertical line through the point $(11, -5)$.



$$\boxed{x = 11}$$

(d)[2] Determine an equation of the line through the point $(11, -5)$ which is parallel to the line $7x - 4y = 1$.

$7x - 4y = 1$

$-4y = -7x + 1$

$y = \frac{7}{4}x - \frac{1}{4}$

$\therefore$ $m = \frac{7}{4}$.

$\therefore$ $y - (-5) = \frac{7}{4}(x - 11)$

$$\boxed{y + 5 = \frac{7}{4}(x - 11)}$$

or $\boxed{y = \frac{7}{4}x - \frac{97}{4}}$

Question 2:

(a) [4] Determine the point of intersection of the following pair of lines:

$$L: \quad 2x - 4y = -8$$
$$M: \quad 3x + 6y = 0$$

Using L: $-4y = -2x - 8$

$y = \frac{1}{2}x + 2$

Now using M: $3x + 6\left(\frac{1}{2}x + 2\right) = 0$

$3x + 3x + 12 = 0$

$6x + 12 = 0$

$x = \frac{-12}{6} = -2$

$\therefore \; y = \frac{1}{2}(-2) + 2 = 1$

$\boxed{\therefore \text{ Point of intersection is } (-2, 1)}$

(b) [6] Eight hundred people attend a basketball game, and total ticket sales are \$3102. If adult tickets are \$6 and student tickets are \$3, how many adults and how many students attended the game?

Let x = number of adults

y = number of students.

① $x + y = 800$

② $6x + 3y = 3102$

Using ①: $y = 800 - x$

using ②: $6x + 3(800 - x) = 3102$

$6x + 2400 - 3x = 3102$

$3x = 702$

$x = 234$

$\therefore \; y = 800 - 234 = 566$

$\boxed{\therefore \text{ 234 adults and 566 students attended the game.}}$

Question 3:

(a)[6] At a price of $p = \$5$ the demand for a certain product is 12,000 units while the supply is 10,000. If price is increased by \$1 demand decreases by 1200 while supply increases by 1000. What is the market price?

Demand line contains point $(5, 12000)$ and has slope -1200.

$\therefore$ equation is $D - 12000 = -1200(p-5)$

$$D = -1200p + 18000$$

Supply line contains point $(5, 10{,}000)$ and has slope 1000.

$\therefore$ equation is $S - 10{,}000 = 1000(p-5)$

$$S = 1000p + 5000$$

Solving $S = D$: $1000p + 5000 = -1200p + 18000$

$$2200p = 13000$$

$$p = \frac{13\cancel{000}}{22\cancel{00}} = \frac{65}{11} \approx \$5.91$$

$\boxed{\therefore \text{ Market price is } \$5.91.}$

(b)[4] Perform one row operation on the following matrix to put it in row echelon form, then determine how many solutions the corresponding system has. The answer is either one, zero, or infinitely many solutions; state which. (Note: you do not have to find the solutions, just say how many there are.)

$$\left[\begin{array}{rrr|r} 1 & 3 & 2 & 0 \\ 0 & 1 & -5 & 2 \\ 0 & -2 & 10 & -4 \end{array}\right]$$

$R_3 = (2)r_2 + r_3$: $\left[\begin{array}{rrr|r} 1 & 3 & 2 & 0 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ Fewer non-zero rows than variables.

$\boxed{\therefore \text{ System has infinitely many solutions.}}$

Question 4:

(a)[5] Ned sells his prize tomatoes at the local farmers market on Saturday mornings. On a particular Saturday he must pay \$100 to rent a space in the market for the day, and he also has to buy a new table at a cost of \$25. Each tomato costs \$0.20 to produce, and he has 160 available to sell. At what price should be sell the tomatoes to ensure that he breaks even? (Round your final answer to two decimal places.)

Let C = total cost of producing and selling 160 tomatoes,
p = selling price of each tomato.
R = revenue from selling 160 tomatoes.

$C = 100 + 25 + (160)(0.20) = \$157$

$R = 160p.$

We need $160p = 157$

$$p = \frac{157}{160} \approx \$0.98$$

$\therefore$ Ned should sell the tomatoes for \$0.98 each to break even

(b)[5] You have \$12,000 to invest and three investments are available to you: one pays 4% per year, the second 5% per year, and the third pays 7% per year. Your goal is to receive an annual income of \$625 per year from the investments, and you require that the amount invested at 7% be equal to the total invested in the other two investments. Set up a system of equations whose solutions would give the amounts to be invested at the different interest rates. DO NOT SOLVE THE SYSTEM OF EQUATIONS, simply set it up. Clearly describe each of the variables used.

Let x = amount invested at 4%.
y = amount invested at 5%
z = amount invested at 7%.

$x + y + z = 12000$

$0.04x + 0.05y + 0.07z = 625$

$z = x + y$

$\therefore$ System is

$$\begin{aligned} x + y + z &= 12000 \\ 0.04x + 0.05y + 0.07z &= 625 \\ x + y - z &= 0 \end{aligned}$$

Question 5 [10]: Solve the following system of equations **using matrix reduction** (no credit will be given for using any other method). Clearly state the row operations used at each step and clearly state the solution set.

$$\begin{aligned} 2x - 3y + 4z &= -15 \\ x - y + z &= -4 \\ 5x + y - 2z &= 12 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 4 & -15 \\ 1 & -1 & 1 & -4 \\ 5 & 1 & -2 & 12 \end{array}\right]$$

$r_1 \leftrightarrow r_2$:
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 2 & -3 & 4 & -15 \\ 5 & 1 & -2 & 12 \end{array}\right]$$

$R_2 = (-2)r_1 + r_2$:
$R_3 = (-5)r_1 + r_3$:
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & -1 & 2 & -7 \\ 0 & 6 & -7 & 32 \end{array}\right]$$

$R_2 = (-1)r_2$:
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 6 & -7 & 32 \end{array}\right]$$

$R_1 = (1)r_2 + r_1$:
$R_3 = (-6)r_2 + r_3$:
$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 5 & -10 \end{array}\right]$$

$R_3 = \left(\frac{1}{5}\right) r_3$:
$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$R_1 = (1)r_3 + r_1$:
$R_2 = (2)r_3 + r_2$:
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\boxed{\therefore\ x = 1,\ y = 3,\ z = -2}$$