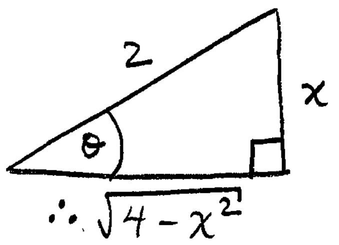
Question 1:

**(a)[4 points]** Simplify $\sec(\arcsin(x/2))$. Your final answer should not contain any trigonometric or inverse trigonometric functions.

Let $\theta = \arcsin\left(\frac{x}{2}\right)$, so $\sin\theta = \frac{x}{2}$.

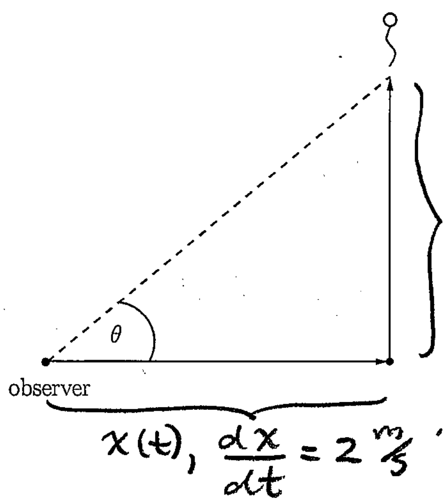



$$\therefore \sec\left(\arcsin\left(\frac{x}{2}\right)\right)$$

$$= \sec(\theta)$$

$$= \boxed{\frac{2}{\sqrt{4-x^2}}}$$

**(b)[6 points]** A balloon released from ground level rises vertically while the wind blows it horizontally away from an observer. At a certain instant in time the balloon is rising at a rate of 3 m/s and is moving horizontally away from the observer at a rate of 2 m/s. At that same instant the balloon is 20 m above the ground and the distance between observer and the point on the ground directly under the balloon is also 20 m. Determine the rate at which the angle of elevation $\theta$ to the balloon is changing at that instant.




$y(t)$, $\frac{dy}{dt} = 3\ \frac{m}{s}$

Find $\frac{d\theta}{dt}$ when $x = y = 20$ m.

$$\theta = \arctan\left(\frac{y}{x}\right)$$

$$\therefore \frac{d\theta}{dt} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{xy' - yx'}{x^2}$$

at $x = y = 20$ : $$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{20}{20}\right)^2} \cdot \frac{(20)(3)-(20)(2)}{(20)^2}$$

$$= \frac{1}{2} \cdot \frac{20}{(20)^2}$$

$$= \boxed{\frac{1}{40}\ \frac{\text{rad.}}{s}}$$

**Question 2:**

**(a)[5 points]** Determine the equation of the tangent line to $y = e^{-3x}\cosh(2x)$ at the point where $x = 0$.

At $x=0$, $y = e^{-3\cdot 0}\cosh(2\cdot 0) = 1$, so $(0,1)$ is a point on the tangent line.

$$y' = e^{-3x}(-3)\cosh(2x) + e^{-3x}\sinh(2x)\cdot 2$$

$$y'\big|_{x=0} = e^{0}\cdot(-3)\cdot\cosh(0) + \cancel{e^{0}\sinh(0)\cdot 2}^{\,0} = -3$$

$\therefore$ Equation of tangent line is $y-1 = -3(x-0)$

or $\boxed{y = -3x+1}$

**(b)[5 points]** Determine $\lim\limits_{x\to\infty}\dfrac{\sinh x}{e^x}$. (L'Hospital's Rule will not be of much help here.)

$$\lim_{x\to\infty}\frac{\sinh x}{e^x} = \lim_{x\to\infty}\frac{\frac{e^{x}-e^{-x}}{2}}{e^x}$$

$$= \lim_{x\to\infty}\frac{1-e^{-2x}}{2}$$

$$= \boxed{\frac{1}{2}}$$

Question 3:

(a)[5 points] Evaluate $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$ $\sim \infty - \infty$

$$= \lim_{x\to 0^+} \frac{\sin x - x}{x\sin x} \sim \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{\cos x - 1}{\sin x + x\cos x} \sim \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{-\sin x}{\cos x + \cos x - x\sin x}$$

$$= \frac{0}{1+1-0}$$

$$= \boxed{0}$$

(b)[5 points] Evaluate $\lim_{x\to 0^+} x^{\sqrt{x}}$ $\sim 0^0$

$$x^{\sqrt{x}} = e^{\sqrt{x}\ln x}.$$

$$\lim_{x\to 0^+} \sqrt{x}\ln x = \lim_{x\to 0^+} \frac{\ln x}{x^{-1/2}} \sim \frac{-\infty}{\infty}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{1/x}{-\frac{1}{2}x^{-3/2}}$$

$$= \lim_{x\to 0^+} -2x^{1/2}$$

$$= 0$$

$$\therefore \lim_{x\to 0^+} x^{\sqrt{x}} = e^0 = \boxed{1}$$

Question 4:

(a) [4 points] Determine the most general antiderivative of $g(x) = \frac{x^3 + 3x^{2/3}}{x^2} - \frac{1}{\sqrt{1-x^2}}$.

$$g(x) = x + 3x^{-4/3} - \frac{1}{\sqrt{1-x^2}}$$

$$\therefore\ G(x) = \frac{x^2}{2} + 3\frac{x^{-1/3}}{-1/3} + \arccos(x) + C$$

$$G(x) = \frac{x^2}{2} - 9x^{-1/3} + \arccos(x) + C$$

(b) [6 points] Determine $f(x)$ if

$$f''(x) = 3\sqrt{x} - \frac{1}{x^2} = 3x^{1/2} - x^{-2}$$
$$f'(1) = 3$$
$$f(1) = 0$$

$$f'(x) = 3\frac{x^{3/2}}{3/2} + x^{-1} + C_1 = 2x^{3/2} + \frac{1}{x} + C_1$$

$$f'(1) = 3 \Rightarrow 3 = 2(1)^{3/2} + \frac{1}{1} + C_1 \Rightarrow C_1 = 0$$

$$\therefore\ f'(x) = 2x^{3/2} + \frac{1}{x}$$

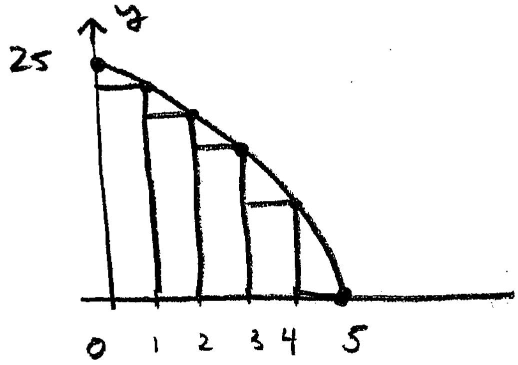
$$\therefore\ f(x) = 2\frac{x^{5/2}}{5/2} + \ln|x| + C_2 = \frac{4}{5}x^{5/2} + \ln|x| + C_2.$$

$$f(1) = 0 \Rightarrow \frac{4}{5}(1)^{5/2} + \cancelto{0}{\ln|1|} + C_2 = 0 \Rightarrow C_2 = -\frac{4}{5}.$$

$$\therefore\ f(x) = \frac{4}{5}x^{5/2} + \ln|x| - \frac{4}{5}$$

Question 5:

(a) **[5 points]** Estimate the area under the graph of $y = 25 - x^2$ from $x = 0$ to $x = 5$ using five approximating rectangles and right endpoints. Along with your answer, state whether it is an overestimate or underestimate of the exact area.




$\Delta x = \frac{5-0}{5} = 1$

$\therefore A \approx R_5$

$= (25-1^2)(1) + (25-2^2)(1) + (25-3^2)(1) + (25-4^2)(1) + (25-5^2)(1)$

$= \boxed{70}$

$R_5$ is an $\boxed{\text{underestimate}}$ of $A$.

(b) **[5 points]** An object travels in a straight line with velocity $v(t) = t\cos^2(\pi t)$ over the time interval $0 \leq t \leq 8$ seconds. Use four subintervals and right endpoints to estimate the total distance traveled during the eight second journey.

total distance = area under graph of $y = v(t)$ over $[0,8]$

$\approx R_4$

$\Delta t = \frac{8-0}{4} = 2$ ; $v(2) = 2$, $v(4) = 4$, $v(6) = 6$, $v(8) = 8$

$\therefore R_4 = (2)(2) + (4)(2) + (6)(2) + (8)(2)$

$= \boxed{40 \text{ m}}$